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The Limits of the Limits of Arbitrage

ALON BRAV[1], J.B. HEATON[2] and SI LI[3]
[1]*Professor of Finance, Duke University Fuqua School of Business;* [2]*Partner, Bartlit Beck Herman Palenchar & Scott LLP;* [3]*Assistant Professor of Finance, Wilfrid Laurier University School of Business and Economics*

Abstract. We test the limits of arbitrage argument for the survival of irrationality-induced financial anomalies by sorting securities on their individual residual variability as a proxy for idiosyncratic risk – a commonly asserted limit to arbitrage – and comparing the strength of anomalous returns in low versus high residual variability portfolios. We find no support for the limits of arbitrage argument to explain undervaluation anomalies (small value stocks, value stocks generally, recent winners, and positive earnings surprises) but strong support for the limits of arbitrage argument to explain overvaluation anomalies (small growth stocks, growth stocks generally, recent losers, and negative earnings surprises). Other tests also fail to support the limits of arbitrage argument for the survival of overvaluation anomalies and suggest that at least some of the factor premiums for size, book-to-market, and momentum are unrelated to irrationality protected by limits to arbitrage.

JEL Classification: G11, G12, G14

1. Introduction

Empirical asset pricing tests of the predictions of the Sharpe-Lintner CAPM often result in model falsification. Small stocks earn returns that are higher than predicted (see Banz, 1981), as do recent winners (see, e.g., Chan et al., 1996), value stocks (see, e.g., Lakonishok et al., 1994), and stocks of companies with positive earnings surprises (see, e.g., Ball and Brown, 1968; Bernard and Thomas, 1990). Growth

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stocks, recent losers, and negative earnings surprises earn returns that are lower than predicted (see, e.g., Ritter, 1991; Barberis and Huang, 2008; Ball and Brown, 1968; Bernard and Thomas, 1990; Chan et al., 1996).

Researchers in behavioral finance argue that asset pricing anomalies result from the influence of unmodeled irrational behavior on security prices (see, for example, the extended discussion in Barberis and Thaler (2003) of the above anomalies). The behavioral claim is controversial. First, rational behavior is just one of several assumptions used to derive the tested asset pricing models. Model falsification might result from the failure of an assumption other than the assumption of rationality (see Cross, 1982; Lowenstein and Willard, 2006; Fama, 1991). Second, the behavioral claim implies that rational arbitrageurs cannot exploit the irrational investors and drive irrationality-induced price deviations to zero (see Friedman, 1953; Fama, 1965) or near-zero (see Grossman and Stiglitz, 1980) levels. We call this the "arbitrage objection."

To rebut the arbitrage objection, some researchers have argued that arbitrage is more difficult than recognized by those offering the arbitrage objection. Barberis and Thaler (2003) survey the literature and identify three sources of limits to arbitrage: idiosyncratic risk, noise trader momentum risk, and implementation costs. Idiosyncratic risk is proposed as a limit to arbitrage in several papers.[1] In their paper, "The Limits of Arbitrage," Shleifer and Vishny (1997) explain their model as follows:

> To specialized arbitrageurs, both systematic and idiosyncratic volatility matters. In fact, idiosyncratic volatility probably matters more, since it cannot be hedged and arbitrageurs are not diversified. ... In our model ... stocks are not rationally priced, and idiosyncratic risk deters arbitrage. ... For a given noise trading process, volatile securities will exhibit greater mispricing and a higher average return to arbitrage.

Barberis and Thaler (2003) explain idiosyncratic risk as a deterrent to arbitrage by using the example of a hypothetically undervalued share of Ford Motor Company:

> The most obvious risk an arbitrageur faces if he buys Ford's stock at [the undervalued price of] $15 is that a piece of bad news about Ford's fundamental value causes the stock to fall further, leading to losses ... Shorting General Motors protects the arbitrageur somewhat from adverse news about the car industry as a whole, but still leaves him vulnerable to news that is specific to Ford – news about defective tires, say.

Noise trader momentum risk, according to Barberis and Thaler (2003), is the risk that irrational beliefs get worse in the direction already distorting prices:

> Noise trader risk ... is the risk that the mispricing being exploited by the arbitrageur worsens in the short run. Even if General Motors is a perfect substitute for Ford, the arbitrageur still faces

[1] See also Pontiff (1996); Ali et al. (2003); Mendenhall (2004); Wurgler and Zhuravskaya (2002); Mashruwala et al. (2006); and Pontiff (2006).

> the risk that the pessimistic investors causing Ford to be undervalued in the first place become even more pessimistic, lower its price even further. Once one has granted the possibility that a security's price can be different from its fundamental value, then one must also grant the possibility that future price movements will increase the divergence.

Finally, Barberis and Thaler (2003) posit that "[w]ell understood transaction costs such as commissions, bid-ask spreads and price impact can make it less attractive to exploit a mispricing."

In this paper, we report the results of a set of tests designed to test the limits of arbitrage argument. Under the hypothesis that greater idiosyncratic volatility protects the existence of greater levels of mispricing, our first set of tests sorts securities on the size of their residual variability from the Fama and French four-factor model (as a measure of idiosyncratic risk) and ask whether financial anomalies increase in magnitude with the amount of residual variability. Under the hypothesis that momentum in the same direction protects greater levels of mispricing, in our second set of tests we form portfolios of value stocks – which, according to some researchers, earn abnormal positive returns because irrational investors extrapolate past bad performance into the future (see, e.g., Lakonishok et al., 1994) – and sort them into those exhibiting negative momentum and those exhibiting positive momentum, testing the prediction that arbitrage against loser-value stocks is riskier and thus should deter arbitrage and allow for the survival of higher abnormal returns to value stocks.

Our first set of tests reject the limits of arbitrage argument for positive return anomalies – small value stocks, value stocks generally, recent winners, and positive earnings surprises – since in each case we do not find the relation predicted by the limits of arbitrage argument. Instead, these anomalies are stronger when idiosyncratic risk is *low*. These results are robust to alternative specifications of idiosyncratic risks and transactions costs. In fact, the high positive correlations between total and residual volatility and other generally accepted measures of limits of arbitrage like the extent of institutional holding, analyst coverage, and stock price level suggest that it is unlikely that any alternative measures of idiosyncratic risk or transactions costs could change our inferences. To reverse our inferences, alternative measures would have to be both different *and* move in the *opposite* direction from current proxies. It is difficult to imagine such alternative measures retaining plausible interpretation as limits of arbitrage. The evidence is not consistent with the limits of arbitrage argument for the survival of irrationality-induced undervaluation of small value stocks, value stocks generally, recent winners, and positive earnings surprises. However, our first set of tests does strongly support the limits of arbitrage argument for the abnormal returns to small growth stocks, growth stocks generally, recent losers, and negative earnings surprises. Negative returns to these anomalies occur only among stocks with high residual variability from the Fama and French four-factor model.

Our second set of tests rejects the limits of arbitrage argument for both undervaluation and overvaluation anomalies. In those tests we ask whether noise trader momentum risk might allow more mispricing of value and growth firms when recent returns have been bad (for value firms) or good (for growth firms). The idea is simple: if value firms are mispriced due to excessive extrapolation of bad performance, those firms that have had very recent poor performance have momentum in the direction of the mispricing and should be riskier to bet against. The same holds true for overpriced growth firms that have been doing well recently. But we find the opposite to be true: value firms that are recent losers have lower returns than value firms that are recent winners, and growth firms that are recent losers do worse than growth firms that are recent winners. Both results are inconsistent with the behavioral null provided by the noise trader momentum risk version of the limits of arbitrage argument.

Finally, we ask whether there is a factor premium for size, book-to-market, and momentum in portfolios comprised of the lowest residual variability stocks. The lowest residual variability stocks not only have low idiosyncratic risk but also have high median prices, high institutional holdings, a larger number of analysts, considerable liquidity, high divided yields, and comprise a large part of the market capitalization of the entire market. It is difficult to argue that limits of arbitrage are meaningful in these stocks. If the factor premiums for size, book-to-market, and momentum were driven entirely by irrationality then we would expect very little covariation with these factors in the lowest residual variability portfolios. Instead, we find a strong role for these factors in explaining the returns to zero cost portfolios of the lowest residual variability stocks.

The rest of the paper is organized as follows. Section 2 discusses earlier attempts to test the limits of arbitrage argument. We present the data and methodology in Section 3. In Section 4, we report the first set of results linking limits to arbitrage and average returns to size, value, momentum, and earnings surprise based strategies. Section 5 provides an additional test based on the profitability of investing in both value and momentum strategies. In Section 6 we ask whether size, value and momentum premia exist even in low limits to arbitrage environment. We conclude the paper in Section 7.

2. Comparison to Studies Finding Support for Limits of Arbitrage Explanations

We reach opposite conclusions than other recent papers that purport to confirm limits of arbitrage explanations of financial anomalies that we reject here, including Ali et al. (2003), Zhang (2006), and Mendenhall (2004). We find that prior studies, by equal-weighting rather than value-weighting returns, by failing to consider separately undervaluation and overvaluation anomalies, and by using research designs with non-implementable trading strategies (high frequency trading of very small cap securities and event-time analysis), are not robust to stronger research design. At the same time, the poor performance of small growth stocks – the dominant overvaluation anomaly – is so strong in high limits to arbitrage environments that tests of whether that poor performance is confined to high limits to arbitrage environments are not particularly sensitive to research methodology.

Ali et al. (2003) argue that the spread in returns between value and growth firms is cross-sectionally correlated with proxies for arbitrage costs such as idiosyncratic risk and conclude that the book-to-market (i.e., value versus growth) effect is likely to be due to market mispricing. Specifically, they rely on the difference between value and growth size-adjusted buy and hold returns to show that this difference is larger when limits to arbitrage are higher. However, their results are driven entirely by the presence of overvaluation (i.e., the underperformance of growth stocks) in high limits to arbitrage environments, not by the presence of undervaluation (i.e. the overperformance of value stocks). That is, by measuring only the *difference* between value and growth returns, they miss the fact that their result is explained entirely by the underperformance of small growth firms, not by relative good performance of value stocks in high limits to arbitrage environments. This will become clear below when we present results separately for value and growth stocks.

Another study, Zhang (2006), focuses on the interaction between profits to momentum strategies and measures of information ambiguity such as firm size, age, analyst coverage, dispersion in analyst forecasts, cash flow volatility, and stock volatility measured as the standard deviation of weekly excess returns relative to the market portfolio measured over the one-year pre formation period. Zhang finds that loser (winner) stocks tend to earn larger negative (positive) abnormal returns in high ambiguity environments that are reasonably interpreted as high limits to arbitrage environments. His results for losers are consistent with ours – losers perform poorly only in high limits to arbitrage environments – but his results for recent winners are inconsistent with both the finding in this paper and those reported in Ang et al. (2006). We find that Zhang's findings hinges primarily on equal weighting his constituent stocks (giving dramatically more weight to the tiniest and relatively illiquid firms than their value-weighted counterparts) and his high frequency rebalancing of such firms that is unlikely to reflect real world trading opportunities. To the extent that equal-weighting can be defended as a way to put more focus on small firms where mispricing might be likely, it still is important to recognize the effect that very small firms with their considerable market microstructure challenges have on measures of abnormal return. In any case, Zhang's results are not robust to value weighting.

Mendenhall (2004) examines the link between arbitrage risk and the post-earnings announcement drift. He finds that both positive and negative drift increase in absolute value as one holds portfolios of high residual variability stocks, a proxy for limits to arbitrage. His evidence is inconsistent with our finding that the positive drift is actually confined to low limit to arbitrage environments. However, while we design our tests around a simple passive and implementable calendar time trading strategy with appropriate adjustment for common risk factors in returns, Mendenhall's design considers only non-implementable event-time analysis, in which returns are equally weighted and risk or factor adjustment is made only with respect to size matched portfolios.[2] Of course, one might argue that an anomaly identified by a non-implementable trading strategy necessarily invokes the limits of arbitrage argument because a trading strategy that is non-implementable is necessarily a limit to arbitrage. This calls into question whether the anomaly is economically interesting since it does not exist in a tradeable form. Moreover, Mendenhall uses a market model to form estimates of residual volatility, leaving common factors in returns due to size, book to market and momentum, which are therefore included in his measure of residual variability. Sorting on his market model residual volatility results in a sort on these firm characteristics. As a result, his control for size related differences in average returns is incomplete.

3. Data and Implementation

We obtain stock returns from the Center for Research in Security Prices for all firms traded on the NYSE, AMEX, and NASDAQ, subject to the restriction that they have CRSP common share codes equal to 10 or 11, from July 1958 to December 2007. Accounting data is obtained from Compustat. We measure the magnitude of specific anomalies with average excess returns, alpha estimates, and Sharpe Ratios (annualized excess returns per unit of standard deviation) for small stocks (including small growth stocks and small value stocks), recent winner stocks, recent

[2] The use of calendar-time regressions to measure abnormal performance has been questioned by Loughran and Ritter (2000). They argue that the implementation of calendar-time regressions can result in low power to reject a null model in tests of market efficiency. One concern is the choice of equal versus value weighting of portfolio returns. If mispricing is likely to occur in hard to arbitrage small firms then a weighting scheme that tilts away from these firms will result in low power. We share this concern and thus report, for each undervaluation and overvaluation anomaly, tests that restrict the sample to the subset of small firms. Loughran and Ritter (2000) also argue that benchmark factors might be "contaminated" by the test assets that are to be priced. For example, the inclusion of the book to market factor, HML, in the Fama and French three-factor model may lead to low power since many of the issuing firms that they try to price are included in this factor. It is unclear, however, why this concern applies in the context of our tests. As we show in Section II, the calendar-time setup affords the power needed to support our conclusions. At any rate, our conclusions do not rest on the three or four -factor models as our tests are benchmarked purposely against the CAPM precisely because we want to measure abnormal returns against a null model that behavioral finance has no objections to.

loser stocks, value stocks, growth stocks, and stocks subject to both positive and negative extreme earnings surprises. For each characteristic-based anomaly (e.g., small stocks) we estimate, beginning in July of 1963, for each firm, a four-factor regression using monthly return data from the preceding five years. We use a minimum of 36 monthly returns to estimate the regressions. Firms are allocated to quartile portfolios based on the magnitude of the estimated residual standard deviation and we value weight firm returns for the ensuing three months. We require a minimum of 50 firms in each portfolio. This portfolio formation is repeated quarterly through December 2007.[3] We estimate idiosyncratic risk from a four-factor asset-pricing model including the Fama and French RMRF, SMB, and HML factors and a momentum factor, MOM. We obtain these factor returns and monthly risk-free rates (which we use to calculate excess returns) from Ken French's web site at Dartmouth College.[4]

4. Results for Characteristic-Based Anomalies

4.1 SMALL STOCKS

Panel A of Table I presents results for small stocks. Small stocks are those traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11 that fall in the bottom size quintile using NYSE market capitalization quintile breakpoints. We allocate these stocks into quartile sorts based on the magnitude of their estimated residual variability. Q1-Low (Q4-High) is a portfolio holding firms with lowest (highest) estimated residual variability.

An immediately interesting result – one that appears throughout our tests – is the apparent existence of large amounts of covariation among stocks with high residual variability. The R-squared of the portfolio of the highest residual variability stocks falls dramatically to 73.51 from 88.06 for the lowest residual variability stocks. It is worthwhile noting that this covariation, if it is not hedgeable, makes betting against high residual variability stocks even riskier since the idiosyncratic risk will not even disappear in large portfolios of potentially mispriced securities.

High returns to small stocks occur only in the lowest idiosyncratic risk quartiles, and not at all in the highest quartile, the opposite of the relation predicted by the limits of arbitrage argument. When we perform CAPM regressions on these portfolio returns, there are no anomalous returns relative to the CAPM for small stocks in high idiosyncratic risk portfolios. The CAPM prices the portfolio of the highest residual variability stocks with an insignificant negative alpha of 34 basis points

[3] None of our conclusions change if portfolios are rebalanced every six or twelve months, although the overall profitability of the momentum strategy (both winners and losers) declines.

[4] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table I. Post-formation results for the "small" stock subsample: 1963–2007

The universe is all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11. We focus on firms in the smallest size quintile. Firm size is the market capitalization in the month of July preceding portfolio formation. Beginning in July of 1963 we estimate, for each firm, a four-factor regression using monthly return data from the preceding five years. We require a minimum of 36 monthly returns. The four factors are the Fama and French RMRF, SMB, HML factors including a momentum factor, MOM. The four factors and the size and book-to-market quintile breakpoints are obtained from Ken French's web site. Firms are allocated into quartiles based on the regression residual standard deviations. We calculate portfolio value weight returns for the ensuing three months requiring a minimum of 50 firms in each portfolio. The sorting procedure is repeated quarterly through October 2007 and we report summary statistics for the period ending in December 2007. The column 'Q1-Low' ('Q4-High') refers to a portfolio holding the securities with the lowest (highest) residual standard deviation. The column 'ALL' refers to the portfolio holding all small stocks. Panel A provides the resulting portfolio return characteristics including CAPM alpha, four-factor factor loadings and alpha, regression $R2$, and number of monthly observations. The remaining rows provide information on firm characteristics allocated to the portfolios in the pre-formation period. Median stock price level is calculated as follows. For each formation quarter, we record the median stock price of firms allocated to a given portfolio as of the preceding month and report the average across the full sample period. Institutional holding is the percent of shares held by institutions calculated by averaging the percent shares held in the pre-formation month. Institutional holdings are from Thomson Financial's CDA/Spectrum Institutional (13f) Holdings data. Number of analysts following a firm is obtained from IBES. We calculate the average number of analysts per firm in the quarter post-formation and then average across firms and across time. Amihud measure is calculated as the ratio of the absolute daily return to daily dollar volume averaged over a month. We average this measure across firms allocated to a portfolio. Percent dollar volume is calculated as follows. We obtain the sum of dollar volume traded of all firms in a given portfolio and calculate the fraction relative to the total dollar volume across all portfolios. Campbell IV is an estimate of stock idiosyncratic volatility calculated as in Campbell et al. (2001). We report the average volatility within a portfolio. Dividend yield is calculated as the difference between the buy and hold return including dividends in the 12 months post-formation and the buy and hold return, over the same period, excluding dividends. We then average these yields across firms in a given portfolio and then across time. Percent of Market Cap is the time series average of the pre formation weight of wealth traded in each portfolio relative to the CRSP universe. In Panels B (C) we further narrow the universe to small growth (value) firms, those in the smallest size quintile and lowest (highest) book-to-market quintile. Book-to market is the ratio of fiscal year-end book equity plus balance sheet deferred taxes in the year preceding the formation year to market equity in December preceding the formation period.

	Q1-Low	Q2	Q3	Q4-High	All
	Panel A: Small Stock Universe				
Average Excess Return	0.90	0.96	0.92	0.38	0.84
Standard Deviation	4.36	6.21	7.69	9.10	6.26
Sharpe Ratio	0.72	0.54	0.41	0.15	0.47
CAPM Alpha	0.54	0.43	0.26	−0.34	0.31
T-stat Alpha	4.32	2.67	1.25	−1.32	1.89
Four-Factor Alpha	0.19	0.02	−0.12	−0.59	−0.09
T-stat Alpha	2.59	0.22	−0.78	−2.70	−0.86
Four-Factor Market Beta	0.77	1.08	1.22	1.30	1.05

Table I. Continued

	Q1-Low	Q2	Q3	Q4-High	All
Four-Factor SMB Loading	0.69	1.01	1.16	1.31	1.00
Four-Factor HML Loading	0.58	0.57	0.34	0.14	0.44
Four-Factor MOM Loading	−0.05	0.01	0.10	0.12	0.08
Four-Factor R2	88.06	90.67	81.93	73.51	87.22
Monthly Observations	533	533	520	524	534
Median Price	15.17	9.21	6.15	3.93	8.02
Institutional Holding	20.59	24.09	18.60	11.18	18.72
Number of Analysts	2.19	2.56	2.64	2.21	2.43
Amihud Measure	4.74	9.29	17.14	34.98	16.19
Percent Dollar Volume	16.76	27.17	29.41	26.66	100.00
Campbell IV	10.25	14.75	18.90	24.38	16.97
Dividend Yield	3.74	1.71	0.88	0.46	1.74
Percent of Market Cap	0.88	0.74	0.59	0.42	2.63
Panel B: Small Growth Universe					
Average Excess Return	0.77	0.66	0.21	−0.68	0.48
Standard Deviation	6.21	8.78	9.31	10.17	8.26
Sharpe Ratio	0.43	0.26	0.08	−0.23	0.20
CAPM Alpha	0.11	−0.21	−0.71	−1.62	−0.26
T-stat Alpha	0.57	−0.71	−2.23	−4.41	−1.11
Four-Factor Alpha	−0.09	−0.37	−0.85	−1.52	−0.46
T-stat Alpha	−0.55	−1.31	−3.38	−3.77	−2.75
Four-Factor Market Beta	1.03	1.33	1.33	1.34	1.22
Four-Factor SMB Loading	0.76	0.85	1.07	1.16	1.11
Four-Factor HML Loading	0.09	−0.08	−0.18	−0.23	−0.06
Four-Factor MOM Loading	0.15	0.22	0.27	0.02	0.14
Four-Factor R2	74.68	63.54	67.85	60.07	76.51
Monthly Observations	381	381	381	381	480
Median Price	12.60	8.02	5.75	3.97	6.97
Institutional Holding	23.84	19.28	13.70	8.79	16.54
Number of Analysts	2.85	2.84	2.46	2.12	2.69
Amihud Measure	3.82	6.78	9.06	17.72	9.14
Percent Dollar Volume	24.65	28.10	25.99	21.62	100.00
Campbell IV	13.92	18.07	20.76	24.48	19.21
Dividend Yield	1.35	0.57	0.39	0.18	0.65
Percent of Market Cap	0.17	0.13	0.11	0.08	0.49
Panel C: Small Value Universe					
Average Excess Return	1.16	0.92	1.08	1.20	1.14
Standard Deviation	4.80	6.30	7.35	8.06	5.90
Sharpe Ratio	0.84	0.51	0.51	0.52	0.67
CAPM Alpha	0.78	0.41	0.48	0.58	0.66
T-stat Alpha	4.95	2.06	2.11	2.21	3.97
Four-Factor Alpha	0.32	−0.06	0.01	0.11	0.16
T-stat Alpha	3.60	−0.50	0.05	0.55	1.84
Four-Factor Market Beta	0.83	1.07	1.23	1.22	1.02
Four-Factor SMB Loading	0.76	1.03	1.11	1.17	0.98
Four-Factor HML Loading	0.77	0.86	0.84	0.70	0.80
Four-Factor MOM Loading	−0.06	−0.06	−0.03	0.01	−0.03
Four-Factor R2	87.20	86.75	81.86	70.33	89.42

Table I. Continued

	Q1-Low	Q2	Q3	Q4-High	All
Monthly Observations	458	454	453	459	534
Median Price	13.22	8.00	5.65	3.82	6.99
Institutional Holding	22.76	24.36	19.44	11.62	19.40
Number of Analysts	2.11	2.34	2.34	2.06	2.23
Amihud Measure	6.31	11.27	22.76	55.57	23.54
Percent Dollar Volume	22.75	27.73	26.28	23.24	100.00
Campbell IV	10.56	14.61	18.79	24.73	17.08
Dividend Yield	3.66	1.80	1.10	0.66	1.85
Percent of Market Cap	0.20	0.16	0.11	0.08	0.54

per month. The lowest idiosyncratic risk portfolio, however, earns a statistically significant positive 54 basis points per month. The same result hold for returns relative to the Fama and French four-factor model. The anomalous returns to the smallest stocks are statistically significant and positive for the lowest idiosyncratic risk portfolio (0.19; t-statistic of 2.59) while the highest idiosyncratic risk quartile earns statistically significant negative returns (−0.59; t-statistic −2.70).

The annualized Sharpe Ratio increases monotonically from the highest idiosyncratic risk quartile to the lowest, contrary to the predictions of the limits of arbitrage argument. The Sharpe Ratio of the lowest idiosyncratic risk quartile is 0.72, while the Sharpe Ratio of the highest idiosyncratic risk quartile is 0.15. The difference between these two Sharpe Ratios is both a return and a standard deviation effect. The highest idiosyncratic risk portfolio has the lowest portfolio average monthly excess return of 0.38 and the highest portfolio monthly standard deviation of 9.10. Portfolio standard deviations decrease monotonically from the highest idiosyncratic risk quartile to the lowest. Since we form these *portfolios* using *individual* security residual variance, this tells us that high residual variance securities tend also to have exposures to common factors in returns that do not wash out in large portfolios. That is, firms with high residual variance tend to share high common variation in return. This is apparent from the pattern of four factor loadings. Market loadings increase monotonically from 0.77 in the lowest idiosyncratic risk quartile to 1.30 in the highest idiosyncratic risk quartile. SMB loadings increase monotonically from 0.69 to 1.31 from the lowest to the highest idiosyncratic risk quartile while the loadings on HML decrease monotonically from 0.58 to 0.14. Loadings on the momentum factor exhibit a small increase from the lowest to the highest idiosyncratic risk quartiles. This pattern is apparent in all of our results: high residual variability securities have high market betas and high SMB loadings.

We next explore the evidence with respect to stock price level, institutional ownership, analyst following, liquidity, and market capitalization. ‘Median stock price level’ provides information on the price level of securities allocated to the portfolios. Each quarter, we record the median stock price of firms allocated to a given portfolio as of the preceding month and report the average across the full sample period. For small stocks, median prices fall from the lowest to the highest idiosyncratic risk quartiles. The median price of the lowest idiosyncratic risk quartile is \$15.17, decreasing to \$3.93 in the highest idiosyncratic risk quartile. We obtain institutional holdings from Thomson Financial's CDA/Spectrum Institutional (13f) Holdings data. 'Institutional Holding' is the percent of shares held by institutions calculated by averaging the percent shares held in the pre-formation month and then averaged across time. For small stocks, institutional holding in the lowest idiosyncratic risk quartile is nearly twice that in the high limits portfolio. Number of analysts following a firm is obtained from IBES. We calculate the average number of analysts per firm in the quarter post-formation and then average across firms and across time. For small stocks, there is no clear pattern in the number of analysts.

Illiquid stocks may be harder to arbitrage. We calculate two measures that proxy for illiquidity. The first, 'Percent dollar volume', is calculated by first obtaining the sum of dollar volume traded of all firms in a given portfolio. Then, for each quartile portfolio, we calculate the fraction of dollar volume relative to the total dollar volume summed over all four portfolios. This percentage is then averaged across time. Our second measure, 'Amihud Measure', is the Amihud (2002) illiquidity measure. We calculate a monthly measure by taking the ratio of absolute value of daily return scaled by daily dollar volume, averaged within a month. For each portfolio we report the average of this measure across the allocated securities in the pre-formation quarter and then across time. For small stocks there is no evidence of a monotonic sort on percent dollar volume. However, illiquidity, proxied by the Amihud measure, increases monotonically from the lowest to the highest limit portfolios.

'Campbell IV' is a monthly estimate of stock idiosyncratic volatility calculated based on daily data for each firm as in Campbell et al. (2001). High residual variability firms are more volatile under this measure as well. Dividend yield is calculated as the difference between the buy and hold return including dividends in the 12 months post-formation and the buy and hold return, over the same period, excluding dividends. We then average these yields across firms in a given portfolio and then across time. Dividend yield falls from low residual variability to high residual variability. Pontiff (1996) explains that dividend yield may lower arbitrage costs by shortening the duration of a position in a mispriced security. Hoberg and Prabhala (2009) also find that idiosyncratic volatility is negatively correlated with dividend yield.[5] Finally, percent of market cap is an estimate of the wealth invested

[5] An alternative proxy for limited arbitrage in the context of anomalies involving overvaluation is the cost of shorting. While we do not have access to such data, the evidence in Geczy et al. (2002) is

in each portfolio. It is the proportion of dollars invested in each portfolio in the pre formation period relative to the full CRSP sample averaged over the entire time period. Sorting on residual variability generates a spread in wealth invested even within this small stock universe. The amount of wealth invested in the high residual variability firms is half that invested in low limits to arbitrage firms.

Our results for small stocks are difficult to interpret, however, since small stocks include both overperforming small value stocks and underperforming small growth stocks. Panel B of Table I presents results for only small growth stocks. We define "growth" as firms in the bottom book to market quintile, using breakpoints for the full universe of firms on the NYSE, AMEX, and NASDAQ. We immediately see evidence confirming the limits of arbitrage argument for small growth stocks since the poor performance of small growth stocks is driven entirely by poor performance in the highest residual variability portfolios.[6] Indeed while the Sharpe Ratio of the highest residual variability subset of small growth stocks is −0.23, the lowest residual variability small growth stocks have a Sharpe Ratio of 0.43. The relative poor performance of high residual variability small growth stocks compared with low residual variability small growth stocks is both a return and standard deviation effect. High residual variability small growth stocks return negative 68 basis points per month with a standard deviation of more than 10% per month. The four factor model cannot explain these poor returns. The alpha is −1.52% per month with a t-statistic of −3.77. The level of institutional holding, stock price level, number of analysts and dividend yield are high in the low residual variability portfolio, Q1-Low, and decline monotonically to the high residual variability portfolio Q4-High. Both illiquidity measures indicate that stocks in Q4-High are more illiquid. Notably, however, the set of small growth stocks that are anomalous against the CAPM and the four-factor model are a tiny subset of the CRSP universe market capitalization, less than 0.10%.

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability small growth portfolio and short the low residual variability small growth portfolio. Under the null of the limits of arbitrage argument, the sign on the small growth regression alpha should be negative, since the negative alpha for small growth stocks should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. This is what we find. The CAPM alpha of the zero cost portfolio regression is −1.73 with a t-statistic of −6.05. The 95% confidence interval is from −2.30 to −1.17. We also calculate the Bayesian posterior odds using the methodology of

that it is possible to borrow shares of firms conducting an IPO, suggesting that these costs may not bind for firms which historically have earned poor returns perhaps due to overvaluation.

[6] This poor performance is consistent with the evidence in Ang et al. (2006) who show that high levels of idiosyncratic variance, measured using daily returns, predicts low subsequent average returns, controlling for an array of firm characteristics.

Table II. Zero cost portfolio regressions: 1963–2007

We form zero cost portfolios that are long the highest residual variability portfolios and short the lowest residual variability portfolios from Tables I, III, IV, and V. The table reports CAPM alphas, t-statistics, and 95% confidence intervals for the CAPM alpha. Posterior odds are calculated as in Zellner and Siow (1979) for the hypothesis that the alpha is greater than zero versus less than zero. The posterior odds are approximately F(t)/F(-t) where t is the t-statistic and F(.) is the cumulative standard normal distribution.

	CAPM Alpha	t-statistic	95% C. I.	Posterior Odds
Small Growth	−1.73	−6.05	−2.30 to −1.17	7.35 E-10
Small Value	−0.21	−0.95	−0.63 to 0.22	0.20
Growth	−1.01	−3.43	−1.59 to −0.43	3.02 E-04
Value	−0.05	−0.22	−0.48 to 0.38	0.71
Winners	−0.23	−0.86	−0.74 to 0.29	0.24
Small Winners	−0.96	−3.59	−1.48 to −0.44	1.68 E-04
Losers	−1.43	−5.13	−1.97 to −0.88	1.46 E-07
Small Losers	−1.27	−4.62	−1.81 to −0.73	1.90 E-06
PEAD: Positive Earnings Surprises (Small Firms)	−0.64	−3.20	−1.02 to −0.25	6.88 E-04
PEAD: Negative Earnings Surprises (Small Firms)	−0.58	−2.75	−1.00 to −0.17	2.98 E-03

Zellner and Siow (1979) who approximate the posterior odds for the test that the mean is greater than zero versus less than zero as (prior odds)×(F(t)/F(-t), where F is the cumulative standard normal distribution, and the prior probability distributions are Cauchy centered on zero and diffuse on the standard deviation. With even prior odds (that is, a prior belief that it is equally likely that the high residual portfolio is lower than the low residual variability portfolio as it is that the opposite is true) the posterior odds for the hypothesis that the high residual variability portfolio earns more than the low residual variability portfolio are vanishingly small at 7.35 E-10. In other words, the odds on the limits of arbitrage argument for small growth stocks are about 1.4 billion to 1 in favor of that hypothesis.

Panel C of Table I presents results for small value stocks. These are firms which are classified in the sorting month as falling in the bottom size quintile and the top book to market quintile. Inconsistent with limits of arbitrage explanations of high returns to small value stocks, the highest CAPM and four factor alphas occur in the lowest residual variability portfolio subset of the small value stocks. Similarly, the Sharpe Ratio of the lowest residual variability portfolio is much higher than the Sharpe Ratio of the highest residual variability portfolio (0.84 versus 0.52). This is largely a standard deviation effect. While the low residual variability portfolio of small value stocks earns 4 basis points less per month relative to the high limits portfolio, the standard deviation of the highest residual variability portfolio is 60% higher (8.06 versus 4.80). The CAPM alpha is larger for low limits small value stocks than high limits small value stocks. It is worth noting that not even the four-factor model can explain low residual variability small value stocks. The four-factor alpha is 0.32 with a t-statistic of 3.60. There is no four-factor anomaly in high residual variability small value stocks.

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability small value portfolio and short the low residual variability small value portfolio. Under the null of the limits of arbitrage argument, the sign on the small value regression alpha should be positive, since the positive alpha generated by small value stocks should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. We do not find support for the limits of arbitrage argument for small value stocks. The CAPM alpha of the zero cost portfolio regression is negative, not positive as expected under the null, at −0.21 with a t-statistic of −0.95. The 95% confidence interval is from −0.63 to 0.22. While the 95% confidence interval contains positive numbers up to 0.22, so that it is possible at that level of significance to fail to reject the limits of arbitrage argument for differences up to that level, the Bayesian posterior odds put the odds on the limits of arbitrage argument at only 0.20 for small value stocks. That is, with even prior odds before the regression, the Bayesian posterior odds inference is 5:1 ([1/0.20]:1) against the hypothesis that the high residual variability small value stocks earn higher returns (as expected under the null of the limits of arbitrage argument) versus the opposite. Put another way, even someone who came to the data with a strong prior belief 5:1 in favor of the limits of arbitrage argument for small value stocks (i.e., 83.3% probability that the limits of arbitrage argument for small value stocks is true, 16.7% probability that it is not) would be left indifferent after seeing the data, with equal parts belief for and against the limits of arbitrage argument for small value stocks.[7]

4.2 VALUE AND GROWTH STOCKS

Panel A of Table III presents results for growth stocks. We define "growth" as firms in the bottom book to market quintile, using breakpoints for the full universe of firms on the NYSE, AMEX, and NASDAQ. Firms are further allocated into quartile sorts based on the magnitude of their estimated residual variability. Q1-Low (Q4-High) is a portfolio holding firms with lowest (highest) residual variability. As with small growth stocks, Panel A demonstrates that the poor performance in growth stocks occurs in the highest residual variability portfolios.

[7] It is possible that the correlation that we document between portfolios' average return and standard deviation is due to skewness in portfolio returns, where positive (negative) skewness leads to estimates of mean and variance that are positively (negatively) correlated; We have tested for this possibility by estimating the average portfolio return on the odd-numbered time-series observations and the standard deviation on the even-numbered observations. This effectively generates estimates from independent samples. We find that the results in this section as well as those below remain qualitatively unchanged.

Table III. Post-formation results for the "growth" and "value" stock subsample: 1963–2007

We begin with the universe of firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11. In Panel A we focus on firms in the lowest book-to-market quintile whereas in Panel B we narrow the universe to firms in the highest book to market quintile. The portfolio formation and resulting characteristics are defined in Table I.

	Q1-Low	Q2	Q3	Q4-High	All
		Panel A: Growth Stock Universe			
Average Excess Return	0.39	0.49	0.28	−0.28	0.43
Standard Deviation	4.54	6.86	8.19	9.69	4.78
Sharpe Ratio	0.30	0.25	0.12	−0.10	0.31
CAPM Alpha	−0.03	−0.13	−0.43	−1.05	−0.07
T-stat Alpha	−0.43	−0.97	−2.37	−4.01	−1.14
Four-Factor Alpha	0.18	0.17	−0.23	−0.82	0.13
T-stat Alpha	2.75	1.47	−1.48	−3.83	2.98
Four-Factor Market Beta	0.91	1.22	1.35	1.36	0.99
Four-Factor SMB Loading	−0.23	0.07	0.43	0.92	−0.14
Four-Factor HML Loading	−0.30	−0.60	−0.59	−0.59	−0.35
Four-Factor MOM Loading	−0.02	0.06	0.14	0.08	0.02
Four-Factor R2	90.41	87.33	82.42	74.48	95.64
Monthly Observations	507	507	507	507	534
Median Price	39.25	26.28	15.58	7.10	20.97
Institutional Holding	51.35	41.73	27.34	13.86	34.27
Number of Analysts	11.79	8.34	5.46	3.44	8.59
Amihud Measure	0.18	1.00	3.76	10.93	3.75
Percent Dollar Volume	56.37	24.50	12.35	6.78	100.00
Campbell IV	7.14	11.07	15.51	21.30	13.62
Dividend Yield	2.43	1.36	0.83	0.41	1.29
Percent of Market Cap	29.73	5.62	1.85	0.67	37.87
		Panel B: Value Stock Universe			
Average Excess Return	0.76	0.81	1.14	1.00	0.82
Standard Deviation	4.39	5.92	7.13	7.94	4.74
Sharpe Ratio	0.60	0.47	0.56	0.44	0.60
CAPM Alpha	0.40	0.27	0.55	0.35	0.40
T-stat Alpha	3.24	1.86	2.82	1.62	3.43
Four-Factor Alpha	−0.03	−0.04	−0.03	−0.14	−0.03
T-stat Alpha	−0.36	−0.32	−0.21	−0.76	−0.36
Four-Factor Market Beta	0.94	1.22	1.29	1.39	1.05
Four-Factor SMB Loading	0.15	0.39	0.93	0.94	0.26
Four-Factor HML Loading	0.81	0.66	0.89	0.80	0.80
Four-Factor MOM Loading	−0.07	−0.12	0.00	−0.03	−0.08
Four-Factor R2	81.65	81.09	80.94	74.67	89.66
Monthly Observations	534	534	534	531	534
Median Price	20.41	12.16	7.38	4.40	9.78
Institutional Holding	32.26	30.66	23.05	13.37	25.28
Number of Analysts	6.70	4.74	3.53	2.80	4.97
Amihud Measure	2.45	6.70	18.25	47.52	18.01
Percent Dollar Volume	56.03	23.66	12.95	7.36	100.00
Campbell IV	8.31	12.54	17.04	23.51	15.18
Dividend Yield	4.95	2.55	1.43	0.77	2.51
Percent of Market Cap	4.87	1.33	0.49	0.20	6.89

The low returns to growth stocks are a phenomenon of the high residual variability portfolio. The high residual variability CAPM alpha is −1.05 percent per month with a t-statistic of −4.01. The four-factor model cannot price high residual variability small growth stocks. The four-factor alpha of the highest residual variability portfolio is −0.82 with a t-statistic of −3.83. The CAPM alpha of the low residual variability portfolio is insignificantly different from zero, while the four-factor alpha is *positive and statistically significant*, partly, it appears, because low residual variability growth stocks earn good returns despite loading negatively on the HML factor. That is, low residual variability growth firms covary strongly with other growth stocks but do not suffer their same poor returns. High residual variability growth stocks covary even more strongly (negatively) with HML but perform much worse than expected under the four factor model given their large market betas and SMB loadings.

The Sharpe Ratio of the highest residual variability subset of growth stocks is −0.10 while the lowest residual variability growth stocks have a Sharpe Ratio of 0.30. The low Sharpe Ratio found in high residual variability growth stocks is both a return and standard deviation effect. High residual variability growth stocks return minus 28 basis points per month, on average, with a standard deviation of about 10% per month. The anomalously poor performing growth stocks comprise less than 3% of the CRSP universe market capitalization.

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability growth portfolio and short the low residual variability growth portfolio. Under the null of the limits of arbitrage argument, the sign on the growth regression alpha should be negative, since the negative alpha for growth stocks should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. This is what we find. The CAPM alpha of the zero cost portfolio regression is −1.01 with a t-statistic of −3.43. The 95% confidence interval is from −1.59 to −0.43. The posterior odds for the limits of arbitrage argument are about 3,000 to 1 ([1/3.02 E-04]:1).

Panel B of Table III presents results for value stocks. These are firms which are classified in the sorting month as falling in the top book to market quintile. The high residual variability CAPM alpha is insignificant. There is no value stock anomaly in high residual variability value stocks. The low residual variability CAPM alpha is 40 basis points per month with a t-statistic of 3.24. The Sharpe ratio of the lowest residual variability portfolio, 0.60, is nearly 40% higher than the Sharpe ratio of the highest residual variability portfolio, 0.44. This is a standard deviation effect. The high residual variability portfolio earns 24 basis points a month more than the lowest residual variability portfolio but its standard deviation is almost twice as high.

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability value portfolio and short the low residual variability value portfolio. Under the null of the limits of arbitrage argument, the sign on the value regression alpha should be positive, since the positive alpha generated by value stocks should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. As for small value stocks, we do not find support for the limits of arbitrage argument for value stocks. The CAPM alpha of the zero cost portfolio regression is negative, not positive as expected under the null, at −0.05 with a t-statistic of −0.22. The 95% confidence interval is from −0.48 to 0.38. While the 95% confidence interval contains positive numbers up to 0.38, so that it is possible at that level of significance to fail to reject the limits of arbitrage argument for differences up to that level, the Bayesian posterior odds put the odds on the limits of arbitrage argument at 0.70 for value stocks. That is, with even prior odds before the regression, the Bayesian posterior odds inference is 1.41:1 against the hypothesis that the high residual variability value stocks earn higher returns (as expected under the null of the limits of arbitrage argument) versus the opposite. Put another way, even someone who came to the data with a prior belief 1.41:1 in favor of the limits of arbitrage argument for value stocks (i.e., 70.5% probability that the limits of arbitrage argument for value stocks is true, 29.5% probability that it is not) would be left indifferent after seeing the data, with equal parts belief for and against the limits of arbitrage argument for value stocks.

4.3 RECENT WINNER AND LOSER STOCKS

Panel A of Table IV presents results for recent winners. We define recent winners as firms whose 11-month buy and hold return leading up to the month prior to the formation period places them in the top quintile of price momentum. Panel A of Table IV demonstrates that the abnormal return in recent winners is *weakest* in the highest residual variability environment. There is no CAPM anomaly for recent winners in high residual variability portfolios: the high residual variability alpha is an insignificant 0.14 basis points per month. The low residual variability CAPM alpha, however, is 36 basis points per month with a t-statistic of 3.44. The Sharpe Ratio of the highest residual variability subset of recent winners is 0.34 while the lowest residual variability recent winners have a Sharpe Ratio of 0.59. This is a standard deviation effect. High residual variability recent winners return 6 basis points more than the lowest residual variability portfolio while the standard deviation of the highest residual variability portfolio is 8.94 compared with 4.80 for the low residual variability portfolio. In Panel B, we restrict the sample to small winners, firms that belong to the bottom size quintile on CRSP. None of conclusions drawn are sensitive to conditioning on firm size.

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability winner (and small winner) portfolio and short the low residual variability winner (and small winner) portfolio. Under the null of the

Table IV. Post-formation results for the “winner” and “loser” stock subsample: 1963–2007

We begin with the universe of all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11. “Winners” (“Losers”) are determined by assigning firms to the highest (lowest) price momentum quintile. Price momentum is measured using past one-year return skipping the pre-formation month. In Panels B and D we further restrict the momentum samples to “Small” firms, those whose market capitalization is in the bottom size quintile. The quartile portfolio formation and resulting characteristics are defined in Table I.

	Q1-Low	Q2	Q3	Q4-High	All
Panel A: Winner Stock Universe					
Average Excess Return	0.81	1.11	1.26	0.87	0.89
Standard Deviation	4.80	6.33	7.50	8.94	5.27
Sharpe Ratio	0.59	0.60	0.58	0.34	0.59
CAPM Alpha	0.36	0.54	0.59	0.14	0.38
T-stat Alpha	3.44	3.57	3.19	0.55	3.68
Four-Factor Alpha	−0.08	0.01	0.13	−0.32	−0.06
T-stat Alpha	−1.20	0.11	0.88	−1.70	−1.02
Four-Factor Market Beta	1.00	1.17	1.29	1.37	1.08
Four-Factor SMB Loading	0.00	0.26	0.50	0.86	0.14
Four-Factor HML Loading	0.06	−0.05	−0.19	−0.16	−0.01
Four-Factor MOM Loading	0.47	0.63	0.64	0.59	0.51
Four-Factor R2	89.91	85.40	82.77	72.73	94.31
Monthly Observations	534	533	534	533	534
Median Price	34.03	24.69	16.49	8.97	20.39
Institutional Holding	39.81	38.59	31.12	18.20	32.22
Number of Analysts	8.15	6.35	5.24	3.61	6.30
Amihud Measure	0.63	1.19	2.36	10.16	3.47
Percent Dollar Volume	41.07	28.32	19.65	10.97	100.00
Campbell IV	8.15	10.83	14.08	19.84	13.18
Dividend Yield	3.39	2.14	1.34	0.66	1.90
Percent of Market Cap	12.80	4.17	1.86	0.70	19.53
Panel B: Small Winner Stock Universe					
Average Excess Return	1.48	1.82	1.59	0.94	1.22
Standard Deviation	4.94	6.92	8.11	9.34	6.97
Sharpe Ratio	1.04	0.91	0.68	0.35	0.60
CAPM Alpha	0.91	1.04	0.70	−0.04	0.72
T-stat Alpha	5.48	4.74	2.68	−0.14	3.75
Four-Factor Alpha	0.31	0.28	−0.06	−0.62	0.03
T-stat Alpha	2.85	1.81	−0.27	−2.35	0.23
Four-Factor Market Beta	0.90	1.14	1.26	1.32	1.14
Four-Factor SMB Loading	0.63	0.83	0.95	1.14	0.91
Four-Factor HML Loading	0.52	0.36	0.28	0.08	0.30
Four-Factor MOM Loading	0.34	0.62	0.67	0.58	0.52
Four-Factor R2	82.32	81.54	76.23	69.68	83.59
Monthly Observations	427	426	426	426	510
Median Price	18.62	13.13	9.55	6.39	11.57
Institutional Holding	22.90	23.70	19.23	12.51	19.50
Number of Analysts	2.32	2.62	2.53	2.31	2.46
Amihud Measure	2.39	3.23	4.67	15.73	6.39

Table IV. Continued

	Q1-Low	Q2	Q3	Q4-High	All
Percent Dollar Volume	17.28	26.07	28.29	28.37	100.00
Campbell IV	11.13	14.22	17.19	22.38	16.19
Dividend Yield	3.15	1.66	0.93	0.51	1.57
Percent of Market Cap	0.25	0.22	0.19	0.14	0.80
Panel C: Loser Stock Universe					
Average Excess Return	0.32	0.20	−0.11	−0.88	0.24
Standard Deviation	6.19	7.68	8.73	9.66	6.30
Sharpe Ratio	0.18	0.09	−0.05	−0.32	0.13
CAPM Alpha	−0.19	−0.44	−0.81	−1.61	−0.33
T-stat Alpha	−1.17	−2.37	−3.46	−5.80	−2.25
Four-Factor Alpha	0.44	0.16	−0.36	−1.17	0.27
T-stat Alpha	4.41	1.12	−2.05	−5.21	3.48
Four-Factor Market Beta	1.08	1.28	1.34	1.33	1.13
Four-Factor SMB Loading	0.04	0.50	0.86	1.11	0.20
Four-Factor HML Loading	0.09	−0.03	0.04	−0.06	0.04
Four-Factor MOM Loading	−0.77	−0.70	−0.59	−0.54	−0.74
Four-Factor R2	89.54	87.83	80.14	74.70	93.03
Monthly Observations	510	507	507	507	534
Median Price	17.29	10.17	6.15	3.45	8.08
Institutional Holding	36.40	29.76	21.24	12.15	25.02
Number of Analysts	7.81	5.60	4.18	3.04	5.96
Amihud Measure	4.14	11.32	21.68	45.11	20.09
Percent Dollar Volume	59.59	22.35	11.92	6.14	100.00
Campbell IV	10.70	15.70	20.26	25.74	18.05
Dividend Yield	3.26	1.66	0.89	0.41	1.61
Percent of Market Cap	8.56	1.83	0.71	0.30	11.39
Panel D: Small Loser Stock Universe					
Average Excess Return	0.56	0.28	0.22	−0.43	0.26
Standard Deviation	6.37	8.05	9.18	10.43	7.55
Sharpe Ratio	0.31	0.12	0.08	−0.14	0.12
CAPM Alpha	0.05	−0.38	−0.51	−1.22	−0.35
T-stat Alpha	0.25	−1.56	−1.81	−3.61	−1.67
Four-Factor Alpha	−0.03	−0.33	−0.46	−0.94	−0.33
T-stat Alpha	−0.30	−2.03	−1.99	−2.84	−2.22
Four-Factor Market Beta	0.98	1.18	1.29	1.29	1.13
Four-Factor SMB Loading	0.93	1.20	1.23	1.37	1.15
Four-Factor HML Loading	0.71	0.57	0.47	0.17	0.49
Four-Factor MOM Loading	−0.43	−0.48	−0.42	−0.47	−0.44
Four-Factor R2	87.01	84.74	73.08	66.89	83.68
Monthly Observations	474	474	474	474	531
Median Price	9.03	5.60	3.88	2.74	4.84
Institutional Holding	23.83	21.27	15.74	9.69	17.78
Number of Analysts	2.47	2.67	2.55	1.94	2.50
Amihud Measure	10.20	19.25	30.64	52.97	27.85
Percent Dollar Volume	25.62	28.76	25.70	19.93	100.00
Campbell IV	14.07	19.06	22.83	27.20	20.74
Dividend Yield	2.33	1.03	0.60	0.33	1.11
Percent of Market Cap	0.21	0.17	0.13	0.09	0.60

limits of arbitrage argument, the sign on the winner and small winner regression alphas should be positive, since the positive alpha generated by winner and small winner stocks should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. As with small value and value stocks, we do not find support for the limits of arbitrage argument for winner or small winner stocks. The CAPM alphas of both zero cost portfolio regression are negative, not positive as expected under the null, at −0.23 with a t-statistic of −0.86 for winners, and −0.96 with a t-statistic of −3.59 for small winners. The 95% confidence interval for winners is from −0.74 to 0.29 and for small winners is −1.48 to −0.44. While the 95% confidence interval for winners (but not small winners) contains positive numbers up to 0.29, so that it is possible at that level of significance to fail to reject the limits of arbitrage argument for differences up to that level, the Bayesian posterior odds put the odds on the limits of arbitrage argument for winners at 0.24. That is, with even prior odds before the regression, the Bayesian posterior odds inference is 4.17:1 against the hypothesis that the high residual variability value stocks earn higher returns (as expected under the null of the limits of arbitrage argument) versus the opposite. The posterior odds for small winners are 1.68 E-04, or more than 6,000 to 1 against the limits of arbitrage argument.

Panel C of Table IV presents results for recent losers. Consistent with our results on other negative anomalies, the worst alphas and Sharpe Ratio are in the highest residual variability portfolio. When we further restrict the sample to small loser firms in Panel D we find again that the conclusions are not sensitive to conditioning on firm size. Table II reports that the posterior odds are overwhelmingly in favor of the limits of arbitrage hypothesis. In unreported results we find that high limits losers are disproportionately small growth firms, consistent with the evidence in Table I, Panel B.[8]

4.4 RESULTS FOR THE POST-EARNINGS ANNOUNCEMENT DRIFT ANOMALY

In this section we extend our analysis and ask whether similar evidence exists in event-study-based anomalous price reactions. In particular, we focus on the well-known post-earnings announcement drift (Ball and Brown, 1968; Bernard and Thomas, 1990), the evidence that stock prices tend to drift in the same direction as the earnings news for three quarters post-event. We begin with the universe of all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10

[8] In unreported analysis, we check whether results in this section are sensitive to industry concentration, namely, whether low or high idiosyncratic risk stocks tend to concentrate in specific industries. We find that low limits to arbitrage stocks tend to have higher representation from industries such as banking, utilities, and financials. However, when we remove these industries from the analysis in this section and repeat our tests we find that our conclusions remain unchanged.

or 11. We then construct decile portfolios sorted based on standardized unexpected earnings ("SUE"). SUE is measured, each quarter, as the difference between a firm's reported earnings and expected earnings where the latter expectations are calculated assuming earnings follow a seasonal random walk with a drift. Beginning in July 1972 we sort firms into deciles based on their most recent quarterly SUE and hold these firms for three months. Following Chordia et al. (2006) we exclude firms whose stock price is below five dollars in the pre-formation month. We check that our sample firms exhibit the same magnitude of abnormal return as documented elsewhere in the literature (e.g., Chordia et al., 2006) by calculating *equal* weight calendar-time portfolio returns for the SUE deciles. A portfolio that is long in the most positive earnings surprise firms and short the most negative surprise firms earns a four factor alpha of 96 basis points, which is similar to that reported in Chordia et al. (2006).[9]

Since our tests are based on calendar-time portfolios in which firm returns are value weighted rather than equal weighted, we present, in Table V, four-factor regression alphas estimated for decile portfolios in which firms are allocated based on their most recently calculated SUE. Firm returns are value weighted and we rebalance quarterly through December 2007. We denote 'D1' through 'D10' portfolios holding securities with the lowest through highest SUEs and report four-factor regression alphas and associated t-statistics for the full sample in Panel A. Once returns are value weighted, the post-earnings announcement drift is less "anomalous," as only the high SUE decile, D10, indicates evidence of economically meaningful mispricing relative to the four-factor model (19 basis points with a t-statistic = 2.24).

Since full sample evidence indicates that most of the drift is driven by small firms, and our goal is to examine the link between potential mispricing and residual variability, we proceed to narrow our sample of firms to those in the lowest size quintile (see Section 4.1) and form SUE quintile portfolios. The regression alphas are now presented in Panel B of Table V. The lowest SUE portfolio underperforms by 77 basis points per month (t-statistic $= -6.79$) and alphas increase nearly monotonically to the high SUE decile whose four-factor alpha is 33 basis points (t-statistic = 2.40).

With this pattern of drift in the extreme portfolios, we proceed, in Panel C, and ask whether the abnormal drift is concentrated in firms with high residual variability. We sort firms independently on both SUE and residual variability. Beginning in July 1972, we determine quintile breakpoints based on lagged firms' SUE and median breakpoints based on residual variability measured in the pre-formation five-year period as described in Table I. We allocate firms into the resulting ten

[9] Chordia et al. (2006)'s sample contains all NYSE and AMEX stocks with prices greater than five dollars and extends over a similar time period as ours, 1972–2004, and they report a four-factor alpha of 65 basis points.

Table V. Post earnings announcement drift: 1972–2007

The universe is all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11. The table provides regression results for decile portfolios sorted based on standardized unexpected earnings ("SUE"). SUE is measured, each quarter, as the difference between a firm's reported earnings and expected earnings where the latter expectations are calculated assuming earnings follow a seasonal random walk with a drift. Beginning in July 1972 we sort firms into deciles based on their most recent quarterly SUE and hold firms for three months. The notation 'D1' through 'D10' refers to a portfolio holding the securities with the lowest through the highest SUEs. Firm returns are value weighted and we rebalance quarterly through December 2007. We exclude firms with prices below five dollars in the pre-formation month. In Panel A we report four factor regression alphas and associated t-statistics for the full sample. The four factors are the Fama and French RMRF, SMB, HML factors including a momentum factor, MOM. The four factors and the size quintile breakpoints are obtained from Ken French's web site. Panel B provides regression results for "small" firms. "Small" firms are identified as firms whose market capitalization is in the bottom size quintile. Firm size is the market capitalization in the month of July preceding portfolio formation. Beginning in July 1972 we sort firms into quintiles based on their most recent quarterly SUE and hold firms for three months. The notation 'Q1' through 'Q5' refers to a portfolio holding the securities with the lowest through the highest SUEs. Panel C provides regression results for the small firm sample where portfolios are sorted independently on both firm SUE and idiosyncratic volatility. Beginning in July 1972 we determine quintile breakpoints based on lagged firm SUE and median breakpoints based on idiosyncratic volatility measured in the pre-formation five-year period as described in Table I. We allocate firms into the resulting ten portfolios and calculate value weight results for the ensuing three months. We rebalance these portfolios quarterly.

Panel A: CRSP Universe

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Alpha	0.08	−0.04	−0.16	−0.14	−0.09	0.00	0.11	0.09	0.08	0.19
T-statistic	0.90	−0.46	−1.90	−1.83	−1.15	−0.02	1.37	1.16	1.05	2.24

Panel B: Small Firm Sample

	Q1	Q2	Q3	Q4	Q5
Alpha	−0.77	−0.41	−0.15	0.13	0.33
T-statistic	−6.79	−3.30	−1.26	1.06	2.40

Panel C: Small Firm Sample: Double Sort

	Q1	Q2	Q3	Q4	Q5
Low IV Alpha	−0.35	−0.11	0.11	0.24	0.62
T-statistic	−3.25	−1.08	0.99	2.11	5.42
High IV Alpha	−0.87	−0.61	−0.31	−0.04	−0.02
T-statistic	−4.72	−2.77	−1.67	−0.24	−0.12

portfolios and calculate value weight results for the ensuing three months. We rebalance these portfolios quarterly. The regression alphas are presented in Panel C of Table V. The first two rows ("Low IV Alpha") provide quintile SUE portfolio alphas and t-statistics for low residual variability stocks. The next two rows ("High IV Alpha") provide alphas and t-statistics for quintile SUE portfolios formed using high residual variability stocks.

Negative drift for extreme negative SUE portfolio is significant in both the high volatility portfolio with a negative 87 basis point alpha (t-statistic = −4.72) and the low volatility portfolio yields a marginally significant alpha of negative 35 basis points (t-statistic = −3.25). This evidence is consistent with the predictions regarding overvaluation that we have discussed earlier. However, we can also see that the positive drift subsequent to positive earnings news is confined entirely to low residual variability portfolios, whereas the high residual variability portfolio alphas are all insignificantly different from zero. The alpha in the extreme positive SUE portfolio is large and significant only in the low volatility portfolio with a 62 basis point alpha (t-statistic = 5.42). The corresponding high volatility portfolio yields an insignificant alpha of negative 2 basis points (t-statistic = −0.12).

Table II presents the results of zero cost portfolio CAPM regressions that are long the high residual variability portfolio of positive earnings surprises for small firms and short the low residual portfolio of positive earnings surprises for small firms. Under the null of the limits of arbitrage argument, the sign on the regression alpha should be positive, since the positive alpha generated by the portfolio of positive earnings surprises for small firms should be strongest in the highest residual variability portfolio if the anomaly is strongest where limits to arbitrage are highest. We do not find support for the limits of arbitrage argument for the portfolios of positive earnings surprises for small firms. The CAPM alpha of the zero cost portfolio regression is negative, not positive as expected under the null, at −0.64 with a t-statistic of −3.20. The 95% confidence interval is from −1.02 to −0.25. The Bayesian posterior odds put the odds on the limits of arbitrage argument at only 0.20 for small value stocks. That is, with even prior odds before the regression, the Bayesian posterior odds inference is 355:1 ([1/2.98 E-03]:1) against the hypothesis that the high residual variability portfolio earns higher returns (as expected under the null of the limits of arbitrage argument) versus the opposite. As with our earlier results, the evidence is consistent with the limits of arbitrage for the negative earnings surprises for small firms, with a CAPM alpha of −0.58, a t-statistic of −2.75, a 95% confidence interval of −1.00 to −0.17, and Bayesian posterior odds of about 1453:1 in favor of the limits of arbitrage argument.[10]

[10] The evidence from recent work on the "accrual anomaly" (see Sloan, 1996) by Lev and Nissim (2006) indicates that our conclusions extend beyond the "anomalies" studied in this paper. Lev and Nissim (2006) show that the inverse relation between accounting accruals and subsequent returns is

5. Profitability of Value Strategies Conditioned on Past Stock Price Momentum

The tests presented in Section 4 were based on predictions from the behavioral literature linking residual variability (and other correlated firm characteristics) to the magnitude of observed anomalies. Consider again the description from Barberis and Thaler (2003):

> Noise trader risk . . . is the risk that the mispricing being exploited by the arbitrageur worsens in the short run. . . . [T]he arbitrageur [] faces the risk that the pessimistic investors causing Ford to be undervalued in the first place become even more pessimistic, lower its price even further. Once one has granted the possibility that a security's price can be different from its fundamental value, then one must also grant the possibility that future price movements will increase the divergence.

Under this hypothesis, the superiority of value stocks should be highest for stocks exhibiting "loser" momentum trends since arbitrageurs, who are aware of these momentum characteristics, are presumably reluctant to purchase value stocks that are more likely to continue to underperform in the short run. Similarly, the underperformance of growth stocks should be stronger for firms exhibiting recent high price appreciation, namely, momentum "winners," because arbitrageurs will not want to bet against growth stocks that are more likely to continue to appreciate in price.

Table VI presents results for these tests. We form four portfolios by sorting firms based on their pre-formation book-to-market and momentum characteristics. The sample is the universe of all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11, further constrained to firms in the bottom quintile by firm size.[11] We form four portfolios requiring that each be allocated a minimum of 50 firms in the formation period. The portfolios are value weighted and rebalanced quarterly. The first portfolio, 'Loser-Value,' holds firms in the bottom momentum quintile and highest book-to-market quintile. The second portfolio, 'Loser-Growth,' holds stocks in the bottom quintiles by both momentum and book-to-market. The third portfolio, 'Winner-Value,' holds stocks in the highest momentum and book-to-market quintiles. The fourth portfolio, 'Winner-Growth,' holds stocks in the highest momentum quintile and bottom book-to-market

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confined to firms with high accruals that, ex-post, tend to underperform standard size and book to market benchmarks (see their Table I and the discussion in Section 6). There is no reliable evidence of undervaluation. Lev and Nissim (2006) also show that high accrual firms are characterized by high residual variability, low market capitalizations, low book to market ratios, low prices, and reduced likelihood of institutional ownership. These firms have precisely the same characteristics of overvalued firms that we find in high limits to arbitrage environments.

[11] We focus on the bottom size quintile since both value and momentum premia are larger for this subset of stocks. We have conducted the same analysis for the full universe and the results remain qualitatively unchanged.

Table VI. Tests of noise trader risk as a limit to arbitrage: 1963–2007

Firms are sorted into portfolios based on their pre-formation book-to-market and momentum characteristics. We begin with the universe of all firms traded on NYSE, AMEX, and NASDAQ with CRSP common share codes 10 or 11. The sample is further constrained to firms in the bottom quintile by firm size. We form four portfolios requiring that each be allocated a minimum of 50 firms in the formation period. The portfolios are value weighted and rebalanced quarterly. The first portfolio, 'Loser-Value,' holds firms in the bottom momentum quintile and highest book-to-market quintile. The second portfolio, 'Winner-Value,' holds stocks in the highest momentum and book-to-market quintiles. The third portfolio, 'Loser-Growth,' holds stocks in the bottom quintiles by both momentum and book-to-market. The fourth portfolio, 'Winner-Growth,' holds stocks in the highest momentum quintile and bottom book-to-market quintile. We report the resulting portfolio return characteristics including CAPM alpha, four-factor factor loadings and alpha, regression R2, and number of monthly observations. The definition of size, momentum and book-to-market are given in Tables I and IV.

Intersection of Momentum and Book to Market Strategies for Small Stocks				
	Loser-Value	Winner-Value	Loser-Growth	Winner-Growth
Average Excess Return	0.66	1.71	−0.28	0.87
Standard Deviation	7.93	6.20	8.58	8.68
Sharpe Ratio	0.29	0.95	−0.11	0.35
CAPM Alpha	0.06	1.12	−1.24	0.07
T-stat Alpha	0.24	5.53	−4.19	0.24
Four-Factor Alpha	−0.08	0.35	−1.04	−0.41
T-stat Alpha	−0.42	2.52	−3.85	−1.40
Four-Factor Market Beta	1.15	1.11	1.23	1.30
Four-Factor SMB Loading	1.20	0.83	1.04	0.87
Four-Factor HML Loading	0.86	0.68	0.02	0.02
Four-Factor MOM Loading	−0.46	0.33	−0.31	0.64
Four-Factor R2	78.43	80.51	65.60	74.58
Monthly Observations	459	433	395	359

quintile. As in earlier results, we report portfolio return characteristics including CAPM alpha, four-factor factor loadings and alpha, regression R^2, and number of monthly observations.

The evidence in Table VI provides a rejection of both of these predictions. Consider the value portfolios first. Under the limits of arbitrage argument, the 'Loser-Value' portfolio should have a higher CAPM alpha than the 'Winner-Value' portfolio because it is more difficult to bet against mispricing in value firms that are continuing to do poorly. But the CAPM alpha of the 'Loser-Value' portfolio is an insignificant 0.06 (t-statistic 0.24 while the CAPM alpha of the "Winner-Value' portfolio is 1.12 with a t-statistic of 5.53. The Sharpe Ratio of the 'Winner-Value' portfolio is 0.95 compared with 0.29 for the 'Loser-Value' portfolio.

The results for the growth portfolios also are inconsistent with the noise trader momentum version of the limits of arbitrage argument. The 'Loser-Growth' portfolio underperforms with a significant CAPM alpha of −1.24 (t-statistic −4.19) while the ‘Winner-Growth’ portfolio has a CAPM alpha of 0.07 (t-statistic 0.24), the opposite of the outcome expected under the noise trader momentum risk null hypothesis.

6. Four Factor Premiums in Low Limits to Arbitrage Environments

In our final set of tests, we ask whether there is a factor premium for size, book-to-market, and momentum in portfolios comprised of the lowest residual variability stocks. The lowest residual variability stocks not only have low idiosyncratic risk but also have high median prices, high institutional holdings, a larger number of analysts, considerable liquidity, high dividend yields, and comprise a large part of the market capitalization of the entire market. It is difficult to argue that limits of arbitrage are meaningful in these stocks. If the factor premiums for size, book-to-market, and momentum were driven entirely by irrationality then we would expect very little covariation with these factors in the lowest residual variability portfolios.

Our previous results already shed some light on this hypothesis: even our lowest limits to arbitrage portfolios load strongly on the SMB and HML factors. To test this more directly, Table VII reports the results of zero cost portfolio regressions where the relevant zero cost portfolio is long one low limits portfolio and short another low limits portfolio. Panel A reports the results of regressions of the returns to the zero cost portfolio that is long low limits small stocks and short low limits large stocks. Each low limit portfolio consists of firms whose residual standard deviation is in the bottom quartile of the respective distribution of pre formation residual standard deviations. The first regression in Panel A is of the returns to that zero cost portfolio on the other known sources of covariation in stocks returns, i.e., the three other factors, *excluding SMB*. The low limits small-minus-big portfolio loads 0.28 on HML with a t-statistic of 3.36 but does not load significantly on RMRF or MOM. There remains an unexplained alpha of 34 basis points per month with a t-statistic of 1.96. This suggests that there is a size premium even in the lowest limits to arbitrage portfolios. The next regression of Panel A adds SMB to the zero cost regression. The loading on SMB is 0.94 with a t-statistic of 9.75. The alpha then declines to an insignificant 10 basis points. Table VIII, panel A, presents the same results for the high limits portfolios. While high limits firms do load more strongly on SMB, there remains a large unexplained negative alpha of -0.71 (t-statistic -3.71), apparently because the high limits small perform much worse than would be predicted by their large loading on SMB.

Panel B of Table VII reports the results of regressions on the zero cost portfolio that is long low limits value stocks and short low limits growth stocks. The first regression in Panel B is on the three other factors excluding HML. The low limits high-minus-low portfolio loads negatively on RMRF, positively on SMB, and

Table VII. Low limits, zero cost portfolio regressions: 1963–2007

Panel A provides regression results for a zero cost portfolio that is long in low limits "small" stocks and short low limits "large" stocks. Small (large) stocks are defined as firms whose market capitalization is in the bottom (top) size quintile in the month of July preceding the portfolio formation. In the first regression we regress the portfolio returns on RMRF, HML, and MOM whereas in the second regression we add SMB to the set of factors. Panel B provides regression results for a zero cost portfolio long in "value" stocks and short "growth" stocks. Value (growth) stocks are defined as firms whose book-to-market ratio is in the top (bottom) book-to-market quintile. In the first regression we regress the portfolio on RMRF, SMB, and MOM whereas in the second regression we add HML to the three factors. Panel C provides regression results for the sample in panel B restricted to "small" firms. Panel D provides regression results for a zero cost portfolio long in recent "winners" and short recent "losers." Winners and losers are defined as firms whose recent one-year buy and hold return is in the top (bottom) momentum quintile. In the first regression we estimate slopes on RMRF, SMB, and HML, whereas in the second regression we add MOM to the three factors. Panel E provides regression results for the sample in panel D restricted to "small" firms. Each low limit portfolio consists of firms whose residual standard deviation is in the bottom quartile of the respective distribution of pre formation residual standard deviations.

	Alpha	RMRF	SMB	HML	MOM	R2	OBS
			Panel A: Small minus Large				
Estimates	0.34	0.08		0.28	−0.06	5.00	533
T-statistics	1.96	1.41		3.36	−0.98		
Estimates	0.10	−0.06	0.94	0.48	0.00	68.19	533
T-statistics	1.05	−1.18	9.75	6.20	0.07		
			Panel B: Value minus Growth				
Estimates	0.51	−0.24	0.20		−0.12	9.41	534
T-statistics	3.07	−4.70	2.97		−1.84		
Estimates	−0.22	0.02	0.38	1.10	−0.05	68.20	534
T-statistics	−2.25	0.76	7.23	24.11	−1.54		
			Panel C: Small Value minus Small Growth				
Estimates	0.92	−0.41	−0.26		−0.15	25.95	458
T-statistics	5.33	−6.09	−1.49		−1.66		
Estimates	0.44	−0.23	−0.14	0.68	−0.11	42.81	458
T-statistics	2.74	−2.41	−0.79	4.81	−1.23		
			Panel D: Winners minus Losers				
Estimates	0.66	−0.17	−0.19	−0.20		3.19	534
T-statistics	2.55	−1.88	−1.54	−1.25			
Estimates	−0.52	−0.09	−0.04	−0.02	1.24	80.00	534
T-statistics	−4.71	−2.25	−0.69	−0.38	30.22		
			Panel E: Small Winners minus Small Losers				
Estimates	1.00	−0.03	−0.18	−0.17		2.52	427
T-statistics	4.84	−0.39	−1.35	−1.52			
Estimates	0.39	−0.05	−0.19	−0.09	0.73	56.18	427
T-statistics	2.66	−1.09	−2.97	−1.29	17.17		

Table VIII. High limits, zero cost portfolio regressions: 1963–2007

This table performs the same tests as in Table VII, using high limits portfolios. Each high limit portfolio consists of firms whose residual standard deviation is in the top quartile of the respective distribution of pre formation residual standard deviations.

	Alpha	RMRF	SMB	HML	MOM	R2	OBS
			Panel A: Small minus Large				
Estimates	−0.38	0.27		0.24	0.00	3.38	524
T-statistics	−1.32	3.13		1.32	0.00		
Estimates	−0.71	0.07	1.35	0.54	0.10	48.87	524
T-statistics	−3.71	0.66	6.40	3.23	0.77		
			Panel B: Value minus Growth				
Estimates	1.56	−0.29	−0.20		−0.19	7.31	531
T-statistics	5.69	−3.37	−1.66		−1.45		
Estimates	0.65	0.03	0.02	1.38	−0.10	40.21	531
T-statistics	2.95	0.47	0.27	12.42	−1.25		
			Panel C: Small Value minus Small Growth				
Estimates	2.22	−0.36	−0.14		−0.09	7.93	459
T-statistics	6.57	−3.71	−0.85		−0.52		
Estimates	1.49	−0.09	0.04	1.04	−0.03	26.62	459
T-statistics	4.70	−0.95	0.24	5.80	−0.21		
			Panel D: Winners minus Losers				
Estimates	1.91	−0.04	−0.40	−0.26		3.81	533
T-statistics	6.50	−0.40	−2.15	−1.28			
Estimates	0.84	0.03	−0.26	−0.10	1.12	47.04	533
T-statistics	3.76	0.45	−2.69	−0.79	12.30		
			Panel E: Small Winners minus Small Losers				
Estimates	1.45	0.04	−0.07	−0.20		1.12	426
T-statistics	5.03	0.35	−0.32	−1.16			
Estimates	0.66	0.02	−0.10	−0.07	0.86	30.36	426
T-statistics	2.70	0.34	−0.77	−0.60	7.47		

negatively on MOM. There remains an unexplained alpha of 51 basis points with a t-statistic of 3.07. This suggests that there is a value-growth premium even in the lowest limits to arbitrage portfolios. The next regression of Panel B adds HML to the zero cost regression. The loading on HML is 1.10 with a t-statistic of 24.11 while the regression intercept is now negative at −0.22 with a t-statistic of −2.25, apparently because the difference in returns to the two portfolios is too small relative to the average return on HML. Table VIII, panel B, presents the same results for the high limits portfolios. While high limits firms do load more strongly on HML, there remains a large unexplained alpha of 0.65 (t-statistic 2.95), apparently because the high limits growth firms perform much worse than would be predicted by the zero cost portfolio's loading on HML. Table VII, panel C, presents the results for only small value minus small growth. There the loading on HML is smaller and there remains a significant alpha of 0.44 with a t-statistic of 2.74. Table VIII, panel C, presents the same results for the high limits portfolios. While high limits small firms do load more strongly on HML, there remains a large unexplained alpha of 1.49 (t-statistic 4.70), again, apparently because the high limits growth firms perform much worse than would be predicted by the zero cost portfolio's loading on HML.

Panel D of Table VII reports the results of regressions on the zero cost portfolio that is long low limits recent winners and short low limits recent losers. The first regression in Panel D is on the three other factors excluding MOM. The low limits winners-minus-losers portfolio loads slightly negatively on RMRF, SMB, and HML. There remains an unexplained alpha of 66 basis points a month with a t-statistic of 2.55. This suggests that there is a momentum premium even in the lowest limits to arbitrage portfolios. The next regression of Panel D adds MOM to the zero cost regression. The loading on MOM is 1.24 with a t-statistic of 30.22. The alpha is now negative at 0.52 with a t-statistic of −4.71, apparently because the zero cost portfolio, while moving strongly with MOM, does not have a sufficiently low return since low limits to arbitrage losers do not underperform. Table VIII, panel D, presents the same results for the high limits portfolios. High limits firms do not load more strongly on MOM, and there remains a large unexplained *positive* alpha of 0.84 (t-statistic 3.76), apparently because the high limits losers firms perform much worse than would be predicted by the zero cost portfolio's loading on MOM. Table VII, panel E, presents the results for only low limits small winner minus small losers. The alpha now becomes positive, apparently because low limits small winners perform much better than would be predicted by the zero cost portfolio's loading on MOM. Table VIII, panel E, presents the same results for the high limits portfolios. The results are qualitatively the same, though the alpha is even larger, apparently because high limits losers perform much worse than would be predicted by the zero cost portfolio's loading on MOM.

7. Conclusions

Behavioral explanations of financial anomalies encounter the objection that irrationality-induced anomalies would be exploited and eliminated by rational arbitrageurs. The limits of arbitrage argument counters this objection by asserting that arbitrage is difficult because idiosyncratic risk and noise trader momentum risk make arbitrage difficult. This argument is testable because it implies that financial anomalies should be strongest where limits to arbitrage (i.e., idiosyncratic risk and noise trader momentum risk) are greatest. We provide those tests in this paper. We find that the limits of arbitrage argument cannot explain the existence of the undervaluation anomalies (high returns to value stocks, recent winners, and positive earnings surprises) but can explain the existence of the overvaluation anomalies (low returns to growth stocks, recent losers, and negative earnings surprises). We find no support, however, for the noise trader momentum risk version of the limits of arbitrage argument. Taken together, these results are a success for behavioral explanations of overvaluation anomalies but present a serious challenge to develop new behavioral explanations for the existence of undervaluation anomalies.

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