



# Risk aversion as risk-neutral pessimism: A simple proof<sup>☆</sup>

J.B. Heaton

Business Law Fellow, The University of Chicago Law School, United States



## ARTICLE INFO

### Article history:

Received 23 May 2018

Received in revised form 17 July 2018

Accepted 29 July 2018

Available online 10 September 2018

### JEL classification:

C65

D84

D91

K14

K15

K41

### Keywords:

Economic analysis of litigation

Risk aversion

Risk neutral probabilities

Pessimism

Criminal law

Plea bargains

Settlement

## ABSTRACT

It has long been understood that risk aversion reduces the likelihood of suit. This article offers a simple proof that risk aversion is observationally equivalent to risk-neutral pessimism by applying a change of measure to risk-neutral probabilities. The observational equivalence of risk aversion to risk-neutral pessimism may provide a useful framework for proving results in the study of several areas of interest, including (1) the role of risk-transfer mechanisms like contingency fees and litigation funding; (2) calls for prohibitions on, or judicial participation in, plea bargains; and (3) broader judicial review of the fairness, reasonableness, and adequacy of settlements in individual (that is, non-class) civil actions.

© 2018 Elsevier Inc. All rights reserved.

## 1. Introduction

Many litigants – especially individual natural persons – are probably risk averse in litigation. Shavell (1982, p. 61) observes that “the general effect of the plaintiff’s risk aversion is to reduce the likelihood of suit” relative to a risk-neutral plaintiff, a fact of risk aversion also analyzed by Viscusi (1988). This article develops this point by offering a simple proof of the observational equivalence of risk aversion and risk-neutral pessimism. For a given risk-averse individual with subjective probabilistic beliefs about a gamble, there is a risk-neutral equivalent individual with a more pessimistic set of probabilities who attaches less weight to good states and more weight to bad states. The proof makes application of a change of measure to risk-neutral probabilities, importing to the economic analysis of litigation a way of conceptualizing risk aversion that is used often in the financial-economic study of derivatives pricing.<sup>1</sup>

## 2. Modeling approach

I prove the observational equivalence of litigant risk aversion and risk-neutral pessimism within the framework of the seminal economic models of the litigation process (Landes, 1971; Gould, 1973; Posner, 1973). Consider a civil litigation for damages and a risk-averse plaintiff and let  $J$  be the size of judgment awarded to the plaintiff if she wins her case. I assume no uncertainty about  $J$ . The true probability that the plaintiff wins her case is unknown and we concern ourselves only with the subjective probabilities held by the plaintiff and the defendant. The plaintiff believes the probability of winning is  $P_p$ , while the defendant believes the probability of plaintiff victory is  $P_d$ . These probabilities of victory include all relevant probabilistic judgments, including the probability of legal error that benefits or costs one side or the other.<sup>2</sup>

We let  $C_p > 0$  be the cost to the plaintiff of taking her case to trial including legal fees, discovery costs, trial exhibits, travel expenses, expert witness fees, court costs, etc., but excluding costs that are not out-of-pocket like opportunity costs of time and attention given to the litigation. We let  $C_d > 0$  be the cost to the defendant, where we

<sup>☆</sup> Many thanks to the editor and an anonymous referee for very helpful guidance.  
E-mail address: [jbheaton@uchicago.edu](mailto:jbheaton@uchicago.edu)

<sup>1</sup> See, for example, Backus et al. (2011, p. 1986) (“In representative-agent models, risk aversion generates risk-neutral distributions that are shifted left (more pessimistic) relative to [the] true distribution.”).

<sup>2</sup> See, for example, Polinsky and Shavell (1989).

are assuming, denoted by the different subscripts, that the plaintiff and the defendant in general will have different litigation costs. We assume that  $C_p$  and  $C_d$  are both common knowledge in the sense that the parties each know the costs that the other will bear in the litigation. We also assume that the defendant is solvent for (has the ability to pay) the judgment  $J$ . The plaintiff has wealth  $W_p$  not including any expected value from the litigation. The plaintiff is risk averse with a concave utility function  $U(\cdot)$ . The defendant is risk neutral.

### 3. Analysis

An individual is risk averse if she always prefers the expected value of an uncertain outcome to the uncertain outcome itself. That is,

$$U(PW_A + (1 - P)W_B) > PU(W_A) + (1 - P)U(W_B), \tag{1}$$

an inequality that holds if and only if the function  $U(\cdot)$  is concave over the relevant wealth levels,  $W_A$  and  $W_B$ , where  $W_A \neq W_B$ . By the intermediate value theorem of calculus, and given  $0 \leq P \leq 1$ , there is a number  $W_C$  where  $W_A < W_C < W_B$  such that  $U(W_C) = PU(W_A) + (1 - P)U(W_B)$ .  $W_C$  is the “certainty equivalent” of the gamble generating  $W_A$  or  $W_B$  with probabilities  $P$  and  $(1 - P)$ , respectively. That is, a person with utility function  $U(\cdot)$  is indifferent between the uncertain prospect of  $W_A$  with probability  $P$  and  $W_B$  with probability  $(1 - P)$  versus the sure thing of  $W_C$ .

Risk-neutral probabilities are “pseudo-probabilities” (that is, not actual subjective probabilities) obtained through a “change of measure” to obtain probabilities that can be used to interpret a certainty equivalent “payoff” as if the person valuing the payoffs was not a risk-averse individual valuing the payoffs at his true subjective probabilities, but a risk-neutral individual valuing the payoffs at the risk-neutral probabilities. “The risk-neutral probabilities coincide with actual subjective probabilities if agents are in fact risk neutral - otherwise they contain in addition adjustments for risk aversion.”<sup>3</sup>

Let  $S$  be the certainty equivalent of the litigation (which can be thought of as the plaintiff’s acceptable settlement amount) such that the plaintiff is indifferent between pursuing the litigation and receiving the certainty equivalent settlement. Then  $S$  satisfies

$$U(W_p + S) = P_p U(W_p + J - C_p) + (1 - P_p)U(W_p - C_p).$$

This implies there is a probability  $\pi$  (which is *not* the plaintiff’s actual subjective probability), such that

$$W_p + S = \pi(W_p + J - C_p) + (1 - \pi)(W_p - C_p)$$

which has the solution

$$\pi = \frac{S + C_p}{J}, \tag{2}$$

It is easy to confirm that  $\pi = \frac{S + C_p}{J}$  satisfies the requirement for a probability that  $0 \leq \pi \leq 1$ , since, as for  $0 \leq \pi$ , all that is required is that  $S, C,$  and  $J$  are each positive, with  $J$  strictly so. As for  $\pi \leq 1$ , risk aversion implies that  $S < (J - C_p)$ , which is sufficient to bound  $\pi$  from above at 1.

We are now ready for our single proposition:

**Proposition 1.** For a risk-averse plaintiff, the risk neutral probability is strictly less than the plaintiff’s subjective probability of victory, that is,  $\pi < P_p$ .

**Proof.** Recall that risk aversion means that the utility of the expected value of the litigation is greater than the expected utility of the litigation, that is,  $U(P_p(W_p + J - C_p) + (1 - P_p)(W_p - C_p)) > P_p U(W_p + J - C_p) + (1 - P_p)U(W_p - C_p)$ . Since  $S$  is the certainty equivalent of (that is, has the same utility as) the expected utility of the litigation,  $P_p U(W_p + J - C_p) + (1 - P_p)U(W_p - C_p), U(P_p(W_p + J - C_p) + (1 - P_p)(W_p - C_p)) > U(S)$  implies that

$$S < P_p(W_p + J - C_p) + (1 - P_p)(W_p - C_p)$$

$\Leftrightarrow$

$$P_p > \frac{S + C_p}{J} = \pi.$$

□

Risk aversion makes a plaintiff act as a risk-neutral plaintiff would act if facing a probability of victory of  $\pi$  less than the actual subjective probability he holds,  $P_p$ , making the risk-averse litigant observationally equivalent to a risk-neutral litigant who is more pessimistic about the litigation.

#### 3.1. Implication and an example

Case merits should drive outcomes in a good justice system. But risk aversion is a characteristic of litigants unrelated to the merits of their disputes. Thus, risk aversion may allow risk-neutral litigants – potentially institutions like governments and large corporations – to benefit by escaping meritorious (in the sense of positive expected value) litigation, or (supposing  $S$  was small relative to  $PJ - C_p$ ) to benefit from low settlements that a risk-neutral litigant would have refused. The result is that some cases are settled as if their merits were much worse. Good cases can settle like bad cases.

Consider an example. A small company that, as a result of the risk aversion of its owners whose wealth is concentrated in ownership of the company, acts as if it is risk averse with natural log utility,  $\ln(\text{enterprise value})$ . The company’s enterprise value before litigation is \$6 million. The company has been the victim of a breach of contract by a large supplier that diminished the company’s enterprise value to this level from a pre-breach value of \$18.5 million. It would cost \$5.4 million to pursue a claim against the breaching supplier. The company’s directors believe the company has a 70% chance of prevailing against the supplier for a judgment of \$12.5 million, all of which will accrue to the enterprise value of the company, returning it to its pre-breach value. The plaintiff’s case therefore has an expected value of \$3.35 million ( $70\% \times \$12.5$  million, the expected judgment, minus \$5.4 million, the costs of litigation). The defendant’s expected value, based on a cost of litigation of \$5 million and the same view as the plaintiff as to the judgment and probability of victory, is \$13.75 million, but the defendant rightly believes that the plaintiff company will not bring the case. The plaintiff’s utility function is such that the expected utility of bringing the case is  $P_p \ln(W_p + J - C_p) + (1 - P_p) \ln(W_p - C_p) = 0.70(\$6 \text{ million} + \$12.5 \text{ million} - \$5.4 \text{ million}) + 0.30(\$6 \text{ million} - \$5.4 \text{ million}) = 11.47 + 3.99 = 15.46 < 15.61 = \ln(\$6 \text{ million})$ , the expected utility of not bringing the case. Because of its risk aversion, the plaintiff company is acting as would a risk-neutral plaintiff that faced  $\pi_N = 0.568 < 0.70 = P_p$ .

### 4. Conclusion

It has been understood since (1982) and Viscusi (1988) that risk aversion reduces the likelihood of suit. This article offers a simple proof that risk aversion is observationally equivalent to risk-

<sup>3</sup> LeRoy (1989, p. 1606). Risk-neutral probabilities are the basis of most financial derivatives pricing models. See, for example, Hull and White (2013, pp. 1364–1365). Here, by contrast with derivatives-pricing models, we assume for simplicity that the pure time value of money is zero. Relaxing this assumption would merely introduce some additional terms without shedding light on the issues at hand.

neutral pessimism by applying a change of measure to risk-neutral probabilities. The observational equivalence of risk aversion to risk-neutral pessimism may provide a useful framework for proving results in the study of several areas of interest, including (1) the role of risk-transfer mechanisms like contingency fees and litigation funding<sup>4</sup>; (2) calls for prohibitions on, or judicial participation in, plea bargains<sup>5</sup>; and (3) broader judicial review of the fairness, reasonableness, and adequacy of settlements in individual civil actions like that which occurs in federal class actions.<sup>6</sup>

## References

Alschuler, A., 1979. Plea bargaining and its history. *Columbia Law Rev.* 79, 1–43.  
 Backus, D., Chernov, M., Martin, I., 2011. Disasters implied by equity index options. *J. Finance* 66, 1969–2012.  
 Gould, J., 1973. The economics of legal conflicts. *J. Legal Stud.* 2, 279–300.

Heaton, J.B., 2018. Litigation Funding: An Economic Analysis. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=3105702](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3105702).  
 Hull, J., White, A., 2013. Credit derivatives. In: Constantinides, G.M., Harris, M., Stulz, R.M. (Eds.), *Handbook of the Economics of Finance*, vol. 2B.  
 Landes, W., 1971. An economic analysis of the courts. *J. Law Econ.* 14, 61–107.  
 LeRoy, S.F., 1989. Efficient markets and martingales. *J. Econ. Lit.* 27, 1583–1621.  
 Polinsky, A.M., Shavell, S., 1989. Legal error, litigation, and the incentive to obey the law. *J. Law Econ. Organ.* 5, 99–108.  
 Posner, R., 1973. An economic approach to legal procedure and judicial administration. *J. Legal Stud.* 2, 399–458.  
 Shavell, S., 1982. Suit, settlement, and trial: A theoretical analysis under alternative methods for the allocation of legal costs. *J. Legal Stud.* 11, 55–81.  
 Spier, K., 2007. Litigation. In: Polinsky, A.M., Shavell, S.M. (Eds.), *The Handbook of Law and Economics*.  
 Spier, K., 2008. Economics of litigation. In: Durlauf, S.N., Blume, L.E. (Eds.), *The New Palgrave Dictionary of Economics*, 2nd ed.  
 Viscusi, W., 1988. Product liability litigation with risk aversion. *J. Legal Stud.* 17, 101–121.

<sup>4</sup> For a review of the economics of contingency fees, see Spier (2007, 2008). For an analysis of the economics of litigation funding, see Heaton (2018).

<sup>5</sup> See, for example, Alschuler (1979). Interestingly, Professor Landes's pioneering work on economic analysis of litigation was directed at the prosecutor-defendant problem. See Landes (1971).

<sup>6</sup> Federal class actions settlements must be reviewed by the court to determine if the settlement is fair, reasonable, and adequate. Fed. R. Civ. P. 23(e)(2). Courts will reject class action settlements that provide only worthless benefits to the class. See, for example, *In re Subway Footlong Sandwich Mktg. & Sales Practices Litig.*, 869 F.3d 551, 556 (7th Cir. 2017).