MISLEADING OMISSIONS: A BAYESIAN FRAMEWORK

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1 INTRODUCTION

Most people understand the idea of lying by omission.[1] In the legal context, many United States jurisdictions recognize different forms of claims for fraudulent omission. Fraudulent omission differs from fraudulent misstatement because it involves the concealment of a material fact rather than an affirmative misrepresentation.[2] Unlike a claim for fraudulent misstatement, a claim for fraudulent omission requires that the one who omitted the material fact had some duty to disclose it.[3] That duty can arise in a number of ways. Under New York law, for example, a party to a business transaction has a duty to disclose an omitted fact where: (1) "the [other] party has made a partial or ambiguous statement, on the theory that once a party has undertaken to mention a relevant fact to the other party it cannot give only half of the truth"; (2) "the parties stand in a fiduciary or confidential relationship with each other"; or (3) "one party possesses superior knowledge, not readily available to the other, and knows that the other is acting on the basis of mistaken knowledge."[4] Liability for omissions of fact can also arise when one offers an opinion knowing facts "that rebut the recipient's predictable inference."[5]

I present here a Bayesian framework for understanding misleading omissions.[6] Bayes' Theorem provides a simple framework for understanding

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1 One legal commentator has reviewed evidence and argues that "[r]esearch suggests that lying by omission may be the most prevalent form of deception." Timothy T. Lau, *Reliability of Present State Impression Hearsay Evidence*, 52 GONZ. L. REV. 175, 192 (2017).

2 Grand Union Supermarkets of the Virgin Islands, Inc. v. Lockhart Realty Inc., 493 F. App'x 248, 254-55
(3d Cir. 2012)("It is generally understood that tortious nondisclosure is a fraud claim based on an omission rather than an affirmative misstatement.") (citations omitted).

3 As put by the securities law professors Sale and Langevoort: "Materiality notwithstanding, there is no automatic duty to disclose wrongdoing or legal risk." Hillary A. Sale & Donald C. Langevoort, *"We Believe": Omnicare, Legal Risk Disclosure and Corporate Governance*, 66 DUKE L.J. 763, 774 (2016).

4 Harbinger Capital Partners LLC v. Deere & Co., 632 F. App'x 653, 656 (2d Cir. 2015) (internal quotations omitted).

5 Omnicare, Inc. v. Laborers Dist. Council Const. Indust. Pension Fund, 135 S. Ct. 1318, 1331 (2015) (comparing liability for omissions in an opinion under Section 11 of the Securities Act with liability under Statement (Second) of Torts Section 539 (1976)).

6 A useful collection of papers on Bayesian methods similar to that used here can be found in Frank Zenker, *Bayesian Argumentation: The Practical Side of Probability*, 1, 7-14 (2013). Bayes' Theorem has a controversial history in legal scholarship, mainly as to the role it should play in deciding cases. *See* Michael O. Finkelstein & William B. Fairley, *A Bayesian Approach to Identification Evidence*, 83 HARV. L. REV. 489, 490, 516-17 (1970); Laurence H. Tribe, *Trial by Mathematics:*

statements like "partial or ambiguous statement," "half of the truth," "mistaken knowledge," and facts "that rebut the recipient's predictable inference," especially in the context of opinions.

Suppose, for example, that a corporate officer made (and believes) the statement, "Based on facts known to me, I believe our conduct is lawful."[7] Suppose that the facts known include the fact that the company had not consulted any lawyer to evaluate the company's conduct.[8] Or suppose that while the corporate officer believes his statement, the facts known to him include that the company's lawyers believe otherwise and that the government is investigating the lawfulness of the company's conduct on suspicion it is not lawful.[9] Intuition tells us that the corporate officer's statement is somehow misleading. But why? As to the first possibility, we might say something like: "Well, the corporate officer might believe that the company's conduct is lawful based on the facts known to him, but I sure would like to have known the company hadn't actually had a lawyer evaluate that conduct." As to the possibility of the company's lawyers' adverse view of the lawfulness of the conduct and the government investigation, we might say: "Shouldn't the officer also have said that the company's lawyers have deep concerns[10] or is under investigation, whatever his own beliefs are?" But why, especially since we assume that the speaking corporate officer really does believe his statement?

Bayes' Theorem helps us understand why the statement is misleading given these other facts, because it allows us to decompose the statement into component parts and then analyze those components. For example, when no lawyer actually conducted an evaluation of the company's conduct, we will see that, not surprisingly, the "facts known to me" have almost no relevance to the statement "our conduct is lawful." The corporate officer's statement, even though believed by the corporate officer, is based almost entirely on the corporate officer's "general" or "prior" beliefs about the probability of the company's conduct being unlawful regardless of these specific facts

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Precision and Ritual in the Legal Process, 84 HARV. L. REV. 1329, 1331, 1338, 1393 (1971). Bernard Robertson & G. A. Vignaux, *Probability – The Logic of Law*, 13 OXF. J. LEG. STUD. 457, 458 (1993) is an excellent overview of probability in law, including Bayes' Theorem. Richard A. Posner, *An Economic Approach to the Law of Evidence*, 51 STAN. L. REV. 1477, 1479 (1999) analyzes the law of evidence through a Bayesian lens. Leading scholars today are increasing exploring the role that Bayes' Theorem can play in analyzing important evidentiary issues like that analyzed here. *See*, for example, Ian Ayres & Barry Nalebuff, *The Rule of Probabilities: A Practical Approach for Applying Bayes' Rule to the Analysis of DNA Evidence*, 67 STAN. L. REV. 1447, 1451-52 (2015); Kristy L. Fields, *Towards a Bayesian Analysis of Recanted Eyewitness Identification Testimony*, N.Y.U. L. REV. 1769, 1771-72 (2013); Yair Listokin, *Bayesian Contractual Interpretation*, 39 J. LEGAL STUD. 359, 360 (2010).

7 *Compare Omnicare*, 135 S. Ct. at 1328-30.

8 *Id.*

9 *Id.*

10 We set aside the problem of waiving attorney-client privilege, but it would be an issue in this example.

known to him. In the case where the corporate officer knows that the company's lawyers believe the company's conduct is unlawful and knows the government suspects the conduct is unlawful, we will see that the corporate officer's prior opinion about the likelihood of the lawfulness of the company's conduct is even more important to his views, since the facts of the company lawyers' opinions and the government's investigation with suspicion of wrongdoing are much less likely to exist if the company's conduct is lawful. We will sort out the elementary math of all this below—just an application of Bayes' Theorem—and doing so will help us understand better what makes statements like these misleading.

After describing the Bayesian framework, we will return to this example, which, as the footnotes describe, comes from discussion in the 2015 opinion of the United States Supreme Court in the *Omnicare* case. The Bayesian framework has straightforward application to securities cases like *Omnicare*. The framework extends to other commercial cases as well, and to cases of consumer fraud and similar claims. I illustrate this with an application to the misrepresentation of the addictive nature of a product, with reference to recent opioid litigation and potentially misleading omissions about addictiveness.

Section 2 sets out Bayes' Theorem in its most basic form, then reinterprets the corporate officer example in that framework. Section 3 presents an application to recent opioid litigation. Section 4 concludes.

2 THE BAYESIAN FRAMEWORK

2.1 A Gentle Introduction to Bayes' Theorem

Bayes' Theorem is a straightforward implication of joint probability, the probability that two things will happen together. Let A and B denote two "things"[11] of some sort. The joint probability of A and B is denoted P (A, B), and is simply the probability, P (.), that both A and B are, exist, or are true. If you imagine a Venn diagram of the probability of all possible things, P (A, B) is the probability intersection of thing A and thing B.

Basic probability theory shows that we can rewrite the joint probability of two things occurring as the probability of one of them existing given that the other one exists, times the probability the other one exists. That is, $P(A, B) = P(A|B)\,P(B)$, where P (. |.) is a "conditional probability" in the sense that it is a probability of the first thing "conditional" or "given that" the second thing (the thing after the vertical bar in the parentheses) exists. P (B) is the probability of B existing across all scenarios, whether or not A

[11] I use "things" here intentionally. It is not necessary to use the jargon of probability theory to understand the gist of Bayes' Theorem in this context.

exists as well. Thus, the probability of A and B existing together can be viewed as asking, first, what is the probability of B happening, P (B), and then, if B exists, what is the probability of A existing, multiplying those probabilities together since both have to happen.

Of course, we can easily switch A and B and it all remains true. That is, P (A, B) = P (A|B) P (B) = P (B|A) P (A). Bayes' Theorem is nothing more than taking these two equivalent ways to express P (A, B): P (A|B) P (B) and P (B|A) P (A), and dividing out one of the P (.) terms (here we'll use P (B)) to give:

$$P\,(A|B) = \frac{P\,(B\;A)\;P\,(A)}{P(B).}$$

That's Bayes' Theorem: the probability of A given B is the probability of B given A times the probability of A, divided by the probability of B.

2.2 *Changing the Labels for Omnicare*

What does this have to do with misleading omissions? Remember our example based on the discussion in *Omnicare*: "Based on facts known to me, I believe our conduct is lawful." We change A to "lawful conduct" and B to "facts I know." We can then interpret "lawful conduct|facts I know" as A|B. Probability can be interpreted (and has a long history of being interpreted)[12] as a degree or an amount of subjective belief.[13] Thus, we can interpret P (.) as the degree of belief in the truth of the thing inside the parentheses. Putting it together, we can interpret the statement "Based on facts known to me, I believe our conduct is lawful" as:

$$P\,(\text{lawful conduct}|\text{facts I know}) > P^*\quad(1)$$

where P (.) denotes probability as a degree of belief and P * is some threshold above which the belief should be held (unless some specific

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12 *See generally* Ward Edwards, Harold Lindman & Leonard J. Savage, *Bayesian Statistical Inference for Psychological Research*, 70 PSYCHOL. REV. 193, 194 (1963) (The earliest exposition of such an interpretation may be found in F. P. Ramsey, *Truth and Probability*, *in* 7 THE FOUNDATIONS OF MATHEMATICS AND OTHER LOGICAL ESSAYS 156-198, (R. B. Braithwaite ed., 1999), though de Finetti's two works around the same time (Bruno de Finetti, *Fondamenti Logici del Ragionamento Probabilistico* [*Logical Foundations of Probabilistic Reasoning*], 9 BOLLETTINO DELL'UNIONE MATEMATICA ITALIANA 258 (1930); Bruno de Finetti, *La Prévision: Ses Lois Logiques, Ses Sources Subjectives* [*Forecasting: Its Logical Laws, Its Subjective Sources*], 7 ANNALES DE L'INSTITUT HENRI POINCARÉ 1, (1937)) was pathbreaking as well. De Finetti's 1974 book, BRUNO DE FINETTI, THEORY OF PROBABILITY: A CRITICAL INTRODUCTORY TREATMENT (1974), is an English-language presentation of his influential views on subjective probability.

13 *See generally* Edwards, Lindman & Savage, *supra* note 12. This work remains a timeless introduction to the basics of probability as subjective belief.

quantification like "there's a 30% chance" is given) to justify the unqualified statement "I believe [something]." For example, we might think of $P^* = .50$ so that a person is entitled to say "I believe x" if $P(x) > 0.5$, that is, if the person believes that x is more likely than not.

We know from above that Bayes' Theorem allows us to reformulate the statement

$$P\text{ (lawful conduct}|\text{facts I know)} \quad (2)$$

into an equivalent representation:

$$\frac{P\text{ (facts I know lawful conduct)}P\text{ (lawful conduct)}}{P\text{ (facts I know).}}$$

2.3 *Analyzing the Omnicare Example*

We can now return to our earlier fact assumptions and evaluate them in terms of Bayes' Theorem. Suppose one of the facts known to the speaking corporate officer is that the company had not consulted any lawyer to evaluate the lawfulness of the company's conduct. That seems to make the corporate officer's statement a "partial or ambiguous statement," "half of the truth," create "mistaken knowledge," or be a fact that "rebuts the recipient's predictable inference." Perhaps this is because the statement implies to a listener that there are facts the corporate officer knows that are important to his belief. Bayes' Theorem allows us to see why the corporate officer's statement is misleading.

Look at the term P (facts I know|lawful conduct). In this example, where the facts known to the corporate officer do not include any facts about any lawyer's evaluation of the lawfulness of the company's conduct, that the conduct is lawful has no obvious relationship to the facts known to the corporate officer. That is, the assumption of lawful conduct as a given may have no tendency to make the facts known to the corporate officer any more or less probable. That means that P (facts I know|lawful conduct) may be about the same as P (facts I know), a probability that is not conditional on lawful conduct. But if

$$P\text{ (facts I know}|\text{lawful conduct)} \approx P\text{ (facts I know)}, \ (4)$$

it follows, because the above terms more or less drop out of the Bayesian reformulation, that we are left with

$$P\text{ (lawful conduct}|\text{facts I know)} \approx P\text{ (lawful conduct)}, \ (5)$$

Which is to say that the corporate officer's opinion about the lawfulness of the company's conduct is essentially independent of the facts he knows, and is based almost entirely on what Bayesian analysts call his "prior" opinion, a belief independent of the facts he has implied are backing it up. It would have been more accurate—perhaps accurate enough not to be misleading—for him to have said, "I don't really know any facts that bear on the issue, but I just think that we are the kind of company whose conduct would be lawful if it were evaluated by a lawyer to see if it was." This becomes clearer when we consider not an absence of any inquiry by a lawyer to evaluate the lawfulness of the company's conduct, but actual knowledge by the corporate officer of facts that make lawful conduct much less likely than unlawful conduct. Suppose the corporate officer knows that the company's lawyers have evaluated the company's conduct and believe it is unlawful and that the government simultaneously is investigating the lawfulness of the company's conduct on suspicion that it is unlawful.

Now look at the probability, P (facts I know|lawful conduct). This next point is key to understanding the Bayesian view: if the company's conduct actually is lawful—something we need not yet know—then these facts known to the corporate officer—i.e., that his company's lawyers believe the conduct is unlawful and that there is a government investigation ongoing on suspicion of wrongdoing—are much less likely than if the company's conduct is unlawful.

That is,

$$P\text{ (facts I know}|\text{lawful conduct)} \ll P\text{ (facts I know}|\text{unlawful conduct)}. \quad (6)$$

We can think in terms of frequencies to aid intuition on this important point: the facts of lawyers who believe the company is engaged in unlawful conduct when a government investigation is ongoing are much more likely to occur at companies where conduct is unlawful than at companies where conduct is lawful. This implies that the following part of our reformulation,

$$\frac{P\ (\textit{facts I know}\,|\,\textit{lawful conduct})}{P\ (\textit{facts I know})} \qquad (7)$$

is small, which requires an even more important role for the corporate officer's prior opinion to override this effect. It would have been more accurate—again, perhaps accurate enough not to be misleading—for him to have said, "I know some pretty bad facts that would be much more likely to be true if our company's conduct is unlawful than if it is lawful, but I really think that we are the kind of company whose conduct is lawful regardless of any particular facts like those pretty bad ones I know." But by saying, "Based on facts known to me, I believe our conduct is lawful," the corporate officer made a statement that ends up being literally true in a very misleading way. P (lawful conduct|facts I know) may be high only because, although P (facts I know|lawful conduct)/P (facts I know) is very small, the term, P (lawful conduct), the corporate officer's prior belief, is very large.

2.4 A Numerical Example

Suppose

$$P\ (\text{facts I know}|\text{lawful conduct}) = 0.20$$
$$P\ (\text{lawful conduct}) = 0.90 \text{ and}$$
$$P\ (\text{facts I know}) = 0.30.$$

Then, applying Bayes' Theorem,

$$P\ (\text{lawful conduct}|\text{facts I know}) = 0.60,$$

which is greater than our assumed threshold of 0.50, even as the statement hides highly material facts.

3 APPLICATION: OPIOID LITIGATION AND ADDICTIVENESS

The securities context of the *Omnicare* case is a natural place to apply the Bayesian view outlined here. But the framework has broader application. Consider statements about the addictive nature of opioids. The opioid crisis is a tremendous and tragic problem.[14] It also has set off a wave of litigation, including claims that manufacturers of prescription opioid medications "overstated the benefits and downplayed the risks of the use of their opioids and aggressively marketed (directly and through key opinion leaders) these drugs to physicians[.]"[15]

Suppose a manufacturer of prescription opioid medications says "We believe that taken as prescribed, opioids aren't addictive." Put in terms of our framework above, we can reformulate this as

$$P\ (\text{opioids aren't addictive}|\text{taken as prescribed}) > P \qquad (8)$$

Bayes' Theorem allows us to reformulate the statement

$$P\ (\text{opioids aren't addictive}|\text{taken as prescribed}) \qquad (9)$$

into an equivalent representation:

$$\frac{P\ (\text{taken as prescribed}|\text{opioids aren't addictive})\ P\ (\text{opioids aren't addictive})}{P\ (\text{taken as prescribed}).} \qquad (10)$$

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14 *See generally* Julie Bosman, *Inside a Killer Drug Epidemic: A Look at America's Opioid Crisis*, N.Y. TIMES, (Jan. 1, 2017), https://www.nytimes.com/2017/01/06/us/opioid-crisis-epidemic.html?smid=pl-share.

15 In re Nat'l Prescription Opiate Litig., 290 F.Supp.3d 1375, 1378 (J.P.M.L. 2017).

There are a number of ways the statement

$$P\text{ (opioids aren't addictive}|\text{taken as prescribed)} \quad (11)$$

can be false or misleading. Most importantly, of course, the manufacturer of prescription opioid medications may simply not believe it. That is, it could be that

$$P\text{ (opioids aren't addictive}|\text{taken as prescribed)} << P^{*} \quad (12)$$

That is the easy case, and not our concern here. But suppose the statement is viewed as an opinion and that it is either true or difficult to prove false. Does that mean it is not misleading? The answer may be no, and the Bayesian framework helps analyze why.

Consider the term P (taken as prescribed|opiods aren't addictive). This term could be fairly large, all else equal. If opioids do not pose a material risk of addiction, but they relieve chronic severe pain, then it is much more likely that opioids are taken as prescribed and not overused. That is likely true even though there are other side effects, like constipation.[16]

Now consider the term P (opioids aren't addictive). We said above that

$$P\text{ (taken as prescribed}|\text{opioids aren't addictive)} \quad (13)$$

may be high. But that may be misleading if, although opioids would be taken as prescribed so long as they aren't addictive, the probability that they aren't addictive is low.

Finally, consider the term P (taken as prescribed). Across all drugs, some patients comply with prescriptions and some don't, and there are many reasons why.[17] There may be a much lower probability of taking medications as prescribed in general than taking medications as prescribed given they aren't addictive and relieve chronic severe pain. We may therefore end up again with a statement that is misleading in the sense that

$$P\text{ (opioids aren't addictive}|\text{taken as prescribed)} \quad (14)$$

may be high, but only because

$$P\text{ (taken as prescribed}|\text{opioids aren't addictive)} \quad (15)$$

16 *See generally* Alfred D. Nelson & Michael Camilleri, *Opioid-Induced Constipation: Advances and Clinical Guidance*, 7 THERAPEUTIC ADVANCES CHRONIC DISEASE 121, 121-134 (2016).

17 *See generally* Jing Jin ET AL., *Factors Affecting Therapeutic Compliance: A Review from the Patient's Perspective*, 4 THERAPEUTICS & CLINICAL RISK MGMT., 269, 269-86 (2008).

Is large, which is misleading because

$$P\text{ (opioids aren't addictive)} \quad (16)$$

Is low, and

$$P\text{ (taken as prescribed)} \quad (17)$$

may also be low relative to P (taken as prescribed|opioids aren't addictive).

It would have been more accurate—again, perhaps accurate enough not to be misleading—for the manufacturer of prescription opioid medications to have said, "We believe that if opioids are taken as prescribed, then opioids aren't addictive, but you probably should know that a lot of people don't take them as prescribed and they likely are quite addictive."

3.1 A Numerical Example

Suppose

$$P\text{ (taken as prescribed}|\text{opioids aren't addictive)} = 0.90$$
$$P\text{ (opioids aren't addictive)} = 0.15\text{/and}$$
$$P\text{ (taken as prescribed)} = 0.25.$$

Then, applying Bayes' Theorem

$$P\text{ (opioids aren't addictive}|\text{taken as prescribed)} = 0.54,$$

which is greater than our assumed threshold of 0.50, even as the statement hides highly material facts.

4 CONCLUSION

"Deception is part of our everyday interactions; it surrounds us in the form of social niceties, misleading statements, wishful thinking, exaggerations, concealment, and flat untruths."[18] Omissions are of considerable interest in the law as well, and the 2015 opinion of the United States Supreme Court in *Omnicare*, a securities case, and other high stakes litigation surrounding misleading omissions, have raised the stakes of better understanding what makes a statement misleading by omission. Bayes' Theorem is a useful structure, especially in the context of opinions for better understanding otherwise loose concepts like "partial or ambiguous statement," "half of the truth," "mistaken knowledge," and facts "that rebut

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[18] Lisa Kern Griffin, *Criminal Lying, Prosecutorial Power, and Social Meaning*, 97 CAL. L. REV. 1515, 1518 (2009).

the recipient's predictable inference." The Bayesian framework has straightforward application to securities cases like that at issue in *Omnicare*. The framework extends to other commercial cases as well, and to cases of consumer fraud and similar claims, like those at issue in opioid litigation.