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Positive Equity Prices with Insolvency Under Legal Solvency Tests

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Abstract

It is well understood that the equity of an insolvent firm can trade for a positive price so long as there is some positive probability that the firm will become solvent at some future point. Currently, however, this insight exists in the case law in an informal sense, while its use in the financial economics literature is highly formalized and not tied to the legal solvency tests that experts, lawyers, and judges must apply in solvency litigation. A simple model of a debtor firm shows why a positive-equity value does not imply solvency under either of two widely-used legal solvency tests. This links a well-known financial economic insight to legal solvency tests. This is of practical importance as market evidence becomes more important in solvency litigation and as directors continue to face important questions of shifting fiduciary duties when the firm becomes insolvent.

I. Introduction

Financial economists understand that the equity of an insolvent company will trade for a positive price so long as there is some probability the firm will become solvent at a future date.[1] More than 40 years ago, Black and Scholes, 1973, and Merton, 1974, demonstrated how a firm's equity can be viewed as a call option on the firm's underlying assets, with the amount of the debt repayment obligation as the strike price. Their basic insight plays an important role in several analytical frameworks concerning default and bankruptcy prediction (see, e.g., Brockman and Turtle, 2003; Galai, Raviv, and Wiener, 2007; Reisz and Perlich, 2007; and Dionne and Laajimi, 2012). In the financial economics literature, however, the important insight that the equity price of an insolvent firm may be positive is buried in highly formal models and not tied to the legal definitions of insolvency that experts, lawyers, and judges must address in litigation and regulatory proceedings.

Where it does appear in case law, the idea that equity can trade for positive prices even when the firm is insolvent is presented informally, usually without linking the observation to the legal solvency test(s) at issue. As one judge put it, "A stumblebum would pay 1 cent for the most hopelessly insolvent firm, as the deal puts none of the bum's nonexistent assets at risk and could pay off if the debtor unexpectedly strikes it rich." (*Covey v. Commercial National Bank of Peoria*, 1992, pg. 661) As the same judge put it almost 20 years later,

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[1]See, e.g., Van Der Wijst (2013, p. 291): "Another phenomenon that is easy to explain in an options context is the positive-equity value of firms or projects that are technically insolvent."

> If a buyer will pay a positive price for the firm's stock, then it is very likely to be solvent. ('Very likely' rather than 'certain' because stock has an option value. Even after a firm is in bankruptcy, its stock will sell for a small price, reflecting the probability that the firm will be reorganized and old equity investors be given some stake in the reorganized firm.) (*Paloian v. LaSalle Bank, N.A.*, 2010, pg. 694)

And as a bankruptcy court put it recently, referring to a plaintiff's expert's explanation,

> equity market cap may simply reflect 'option value' created by investors who think there is little to lose (because the stock price is low) and much to gain (if, against all expectations, the stock price skyrockets), rather than any actual present-day value. (*Tousa*, 2009, pg. 827)

In *Aéropostale, Inc.*, 2016, pg. 394, "Professor Pritchard opined that based on his review, Aéropostale's stock had only option value by December 2015 and needed 'a miracle to turn it around.'"

In this article, a simple model of a debtor firm is used to prove why a positive-equity value does not imply solvency under either of two widely-used legal solvency tests: (1) a test of whether the value of a legal entity's assets exceeds the face value of its liabilities (the balance-sheet solvency test), and (2) a test of whether a legal entity reasonably can be expected to pay its debts as they come due (the ability-to-pay solvency test, sometimes referred to as cash-flow solvency or equitable solvency). The paper's contribution is to link a well-known insight from financial economics to the legal solvency tests that matter in litigation and regulatory proceedings. This is of importance as market evidence becomes more important in solvency litigation (see, e.g., Schwartz and Bryan, 2012) and as directors continue to face important questions of shifting fiduciary duties when the firm becomes insolvent (see, e.g., Baker, Butler, and McDermott. 2008).[2]

II. Solvency Tests

Two solvency tests appear in bankruptcy and corporate law. First is a test of whether a firm reasonably can be expected to pay its debts as they come due (the ability-to-pay solvency test, sometimes referred to as cash-flow solvency or equitable solvency). Second is a test of whether the fair value of a firm's assets exceeds the face value of its liabilities (the balance-sheet solvency test, performed on either a going-concern or liquidation basis). Courts typically describe solvency as a question of fact.[3]

[2]See also *N. Am. Catholic Educ. Programming Found., Inc. v. Gheewalla*, 2007, pg. 101 "[T]he creditors of an insolvent corporation have standing to maintain derivative claims against directors on behalf of the corporation for breaches of fiduciary duties."

[3]See, e.g., *Weinberg*, 2009, pgs. 27-28 "The bankruptcy court's determinations regarding insolvency resolve questions of fact which are reviewed for clear error."

The ability-to-pay solvency test asks if the firm can reasonably expect to pay its debts as they come due. In the federal fraudulent conveyance statute, for example, the question is whether the firm "intended to incur, or believed that [it] would incur, debts that would be beyond [its] ability to pay as such debts matured." (Title 11, U.S.C.A., 548(a)(1)(B)(ii)(III), 2017) The Uniform Fraudulent Transfer Act states the test similarly, whether the transferor "intended to incur, or believed or reasonably should have believed that the debtor would incur, debts beyond the debtor's ability to pay as they became due."(Uniform Fraudulent Transfer Act, 4(a)(2)(ii), 1984) The Uniform Fraudulent Conveyance Act (1918), still the law in New York, asks whether the debtor "intends or believes that he will incur debts beyond his ability to pay as they mature." (Section vi) It is a forward-looking test: it is not enough to be able to meet current obligations; the firm must be able to meet its future obligations as well.

The balance-sheet solvency test asks a different question: is the value of this firm's assets greater than the face value of its liabilities? When the United States Bankruptcy Code defines the word "insolvent" it calls it the "financial condition such that the sum of such entity's debts is greater than all such entity's property, at a fair valuation." (Title 11, U.S.CA., 101(32)(A)). Fair valuation, in practice, means the application of standard valuation methods, including discounted cash flow and multiples-based valuation for going-concern businesses. Debts are valued at face value for purposes of the balance-sheet solvency test,[4] contingent liabilities are discounted for their probability of occurrence,[5] and a positive accounting book balance does not imply solvency.[6]

III. A Simple Model

The model posits a firm that owes debt to a creditor and will generate cash that will either be sufficient to pay the debt or not. There are two periods, time $t = 0$ ("today") and time $t = 1$ ("tomorrow"; the date when the cash flows are received and when the debt is due). Investors (who put the price on the asset, debt, and equity in the model) are risk neutral, so they value firms at the expected value of their possible future cash flows. This assumption saves concern with the effects of risk aversion or risk-seeking behavior and making

[4]See, e.g., *Hanna v. Crenshaw*, 2003, pg. 3 holding that "for purposes of determining whether a debtor is insolvent under section 547, the liabilities of the debtor must be valued at face value."
[5]See *Xonics Photochemical, Inc*., 1988, and *Covey v. Commercial Nat'l Bank of Peoria*, 1992.
[6]See, e.g., *DeRosa v. Buildex Inc*., 1985, pg. 849 "Asset values carried on a balance sheet, even if derived in accordance with 'generally accepted accounting principles,' do not necessarily reflect fair value: 'Generally accepted accounting principles' are not synonymous with any specific [valuation] policy."'

other adjustments that risk preferences necessitate.[7] The risk-free rate is zero, which is a simplifying assumption that allows for skipping the extra steps of discounting future amounts at a discount factor other than 1.

There are two possible states of the world at time $t = 1$, a good state with cash flow G and a bad state with cash flow B. The probability of the good state is p_G and the probability of the bad state is $1 - p_G$ where, of course, $0 \leq p_G \leq 1$ since p_G is a probability bounded between 0 and 1. The value of the assets at time $t = 0$ (remembering that investors are risk neutral) is the probability-weighted sum of the good and bad cash flows:

$$V_A = p_G G + (1 - p_G)B \tag{1}$$

The firm has debt with a face value at time $t = 0$ of D that is due in amount D at time $t = 1$. The creditor is entitled to full payment of D or the entire cash flow in the state, whichever is smaller. That is, the firm must pay its debt in full if it has the cash to do so, or must pay all cash to the creditor if the cash flow it has is below D. The firm's equity receives the amount, if any, remaining after the repayment of debt. The value of the equity at time $t = 0$ is

$$V_E = p_G max\{G - D, 0\} + (1 - p_G)max\{B - D, 0\} \tag{2}$$

In words, the value of the equity is the probability of the good state times either zero or whatever is left over from G after paying D, whichever is bigger, plus the probability of the bad state times either zero or whatever is left over from B after paying D, whichever is bigger. That is, if $G - D$ or $B - D$ is less than zero—because there was not enough money to pay the debt in full in that state of the world—the limited liability of equity means that the equity holder simply receives nothing in that state. The value of the equity cannot go negative.[8]

Although not necessary for the proofs, we define the value of debt for completeness. The value of the debt at time $t = 0$ is

$$V_D = p_G min\{G, D\} + (1 - p_G)min\{B, D\} \tag{3}$$

In words, the value of the debt is the probability of the good state times the minimum of G and D (it will be D if G is big enough to pay the debt and leave money left for equity, and G if that is less than D) plus the probability of the bad state (which is just one minus the probability of the good state) times the

[7]The effect of risk aversion would, of course, depend on the nature of the risk aversion and whether the risk of the cash flows posited in the model was "priced" in equilibrium, that is, whether investors cared about that risk because it was systematic and could not be diversified away. See, e.g., Sharpe, 1964. If the debt and equity in the model were "risky" in a way investors were averse to, then typical models of market equilibrium would predict prices of the debt and equity that were lower than the prices reflected by simple expected values. All else equal, that would tend to reduce the prevalence of positive-equity prices for insolvent firms, but perhaps not by much. If certain firms became solvent in states of the world where risk adverse investors valued income more, the price of equity of insolvent firms would be larger than in the model here.

[8]Of course, the value of equity need not be strictly positive; it could be exactly zero. This would occur when the cash flow in the good state, G, while greater than B, was at or below the face amount of debt, D.

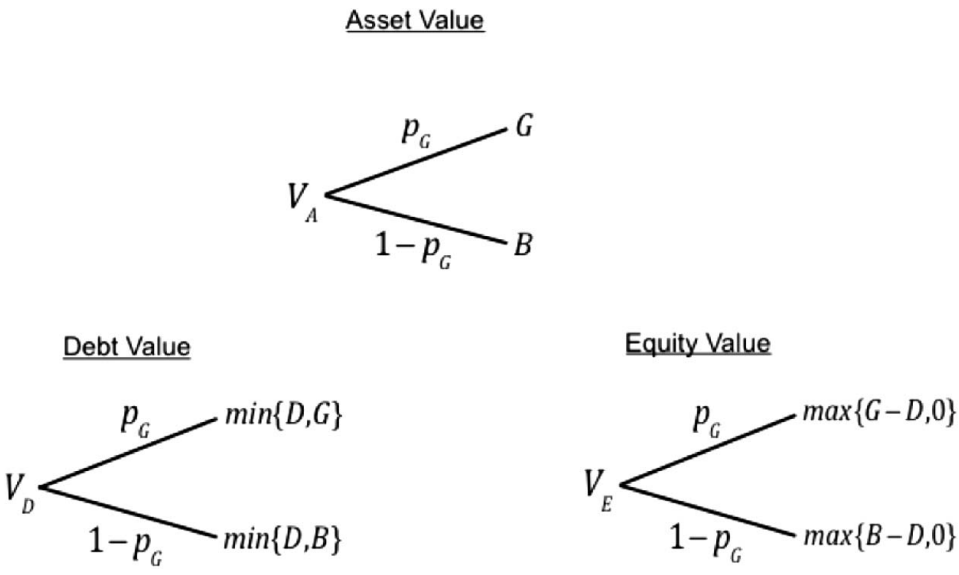

Figure 1. Model of Debtor Firm

minimum of B and D Cases where the good state has cash flow to pay the debt, but the bad state does not, are considered here.

Figure 1 illustrates the simple model of the debtor firm.

Recall that there are two legal solvency tests: (1) a test of whether the value of a legal entity's assets exceeds the face value of its liabilities (the balance-sheet solvency test), and (2) a test of whether a legal entity reasonably can be expected to pay its debts as they come due (the ability-to-pay solvency test). Balance-sheet solvency and ability-to-pay solvency are defined as follows:

Definition of Balance-Sheet Solvency: The debtor firm is *balance-sheet solvent* at time $t = 0$ if, and only if, $V_A \geq D$ The firm is *balance-sheet insolvent* if $V_A < D$.

Definition of Ability-to-Pay Solvency: The debtor firm is *ability-to-pay solvent* at time $t = 0$ if and only if

$$p_G I_{G \geq D} + (1 - p_G) I_{B \geq D} \geq p_S \tag{4}$$

where $I_{G \geq D}$ is an indicator function taking the value of 1 if $G \geq D$ and 0 otherwise, $I_{B \geq D}$ is an indicator function taking the value of 1 if $B \geq D$ and 0 otherwise, and p_S is the minimum probability by which a firm must be expected to pay its debt D in full to be considered ability-to-pay solvent. This probability is not specified here or in case law, but a concept such as "more likely than not" would mean that $p_S > 0.5$. The firm is *ability-to-pay insolvent* if $p_G I_{G \geq D} + (1 - P_G) I_{B \geq D} < p_S$.

The idea behind this definition of the ability-to-pay solvency test is that what matters is total probability of the states of the world where the firm pays D in full when it comes due. If $G \geq D$, then the firm can pay D in full in the good state, otherwise not. If $B \geq D$, then the firm can pay D in full in the bad state,

otherwise not. The indicator function takes the value 1 when the firm can pay its debts in full in that state, and 0 otherwise. This then allows for simple sum of the probabilities where the debt is paid in full.

IV. Solvency Tests and Positive Equity Values

While it is well understood among financial economists that the equity of even an insolvent company may trade for a positive price so long as there is some probability that the firm will become solvent at some future date, that insight derives from models that generally consider the firm to be in default under conditions that do not mirror legal solvency tests applied in, for example, fraudulent transfer litigation.[9] The goal of this section is to establish two basic propositions in the context of legal solvency tests. First, the section establishes the proposition that positive-equity value does not imply balance-sheet solvency. Second, the section establishes the proposition that positive-equity value does not establish ability-to-pay solvency.

Consider the first proposition.

Proposition 1: A positive-equity value does not imply balance-sheet solvency. That is, $V_E > 0 \nRightarrow V_A \geq D$.

Proof: Suppose $B < D < G$ so that the firm is unable to pay its debt in the bad state but can pay the debt in the good state with some amount left for equity. Suppose also that $1 > p_G > 0$, so that there is at least some positive probability of the good state and the bad state. Then the equity value

$$V_E = p_G max\{G - D, 0\} + (1 - p_G) max\{B - D, 0\} = p_G(G - D) > 0.$$

From the definition above, balance-sheet solvency requires

$$p_G G + (1 - p_G)B \geq D \Leftrightarrow p_G G \geq [p_G D + (1 - p_G)D] - (1 - p_G)B \Leftrightarrow p_G G \geq p_G D + (1 - p_G)(D - B) \Leftrightarrow G \geq D + \Big((1 - p_G)/p_G\Big)(D - B).$$

Therefore, the firm has a positive-equity value and is balance-sheet insolvent when $D < G < D + ((1 - p_G) / p_G)(D - B)$, that is, when G falls in a range between D and $D + ((1 - p_G) / p_G)(D - B)$. This proves that a positive-equity value does not imply balance-sheet solvency.[10] This range will be larger, the larger is $(1 - p_G)$ and the larger is $(D - B)$.

Example 1: Suppose $G = 100$, $B = 50$, $p_G = 0.4$, and $D = 75$. Then $V_A = 70$, $V_D = 60$, and $V_E = 10$. The value of the equity is positive, but the debtor is balance-sheet insolvent, since $V_A = 70 < 75 = D$.

[9]See, e.g., *Tronox Inc.*, 1991.

[10]Note that the converse is technically true as well, balance-sheet solvency does not imply positive-equity value, at least in a special case. In the case where $V_A \geq D$ but $G = D$, the firm is solvent but the equity will not have a positive value because, if $B < G$, there is no state of the world where the debt is repaid leaving a surplus for equity.

Now, consider the second proposition.

Proposition 2: A positive-equity value does not imply ability-to-pay solvency. That is, $V_E > 0 \nRightarrow p_G I_{G \geq D} + (1 - p_G) I_{B \geq D} \geq p_S$

Proof: Suppose $B < D < G$ so that the firm is unable to pay its debt in the bad state but can pay the debt in the good state with some amount left for equity. Suppose also that $0 < p_G < p_S$ so that there is at least some positive probability of the good state but the probability of the good state is not as high as p_S. Then the equity value

$$V_E = p_G max\{G - D, 0\} + (1 - p_G) max\{B - D, 0\} = p_G(G - D) > 0.$$

but

$$p_G I_{G \geq D} + (1 - p_G) I_{B \geq D} = p_G I_{G \geq D} = p_G < p_S,$$

so the firm is ability-to-pay insolvent. This proves that a positive-equity value does not prove ability-to-pay solvency.[11]

Example 2: As in Example 1, suppose $G = 100$, $B = 50$, $p_G = 0.4$, and $D = 75$. Then $V_E = 10$ so the value of the equity is positive, but if $p_S = 0.51$, that is, if the firm must be at least 51% likely to pay its debts in full to be considered ability-to-pay solvent, then the debtor firm is ability-to-pay insolvent, since there is only a 40% chance the firm will pay its debts in full.

V. Conclusion

The goal of this short paper was to show that the well-understood fact from option pricing theory—that equity can trade at a positive value even for an insolvent firm—is true under both legal solvency tests, the balance-sheet solvency tests and the ability-to-pay solvency test, even though neither test is the concept of insolvency used in the financial economic models on which the insight rests. Using a simple model of a debtor firm, the paper proves why a positive-equity value does not imply solvency under either of the two widely-used legal solvency tests.

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[11]Note that the converse is technically true as well, ability-to-pay solvency does not imply positive-equity value, at least in a special case. If $p_G > p_S$, $B < G$, and $G = D$,the firm is ability-to-pay solvent but the equity will not have a positive value because, if B < G, there is no state of the world where the debt is repaid leaving a surplus for equity.

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